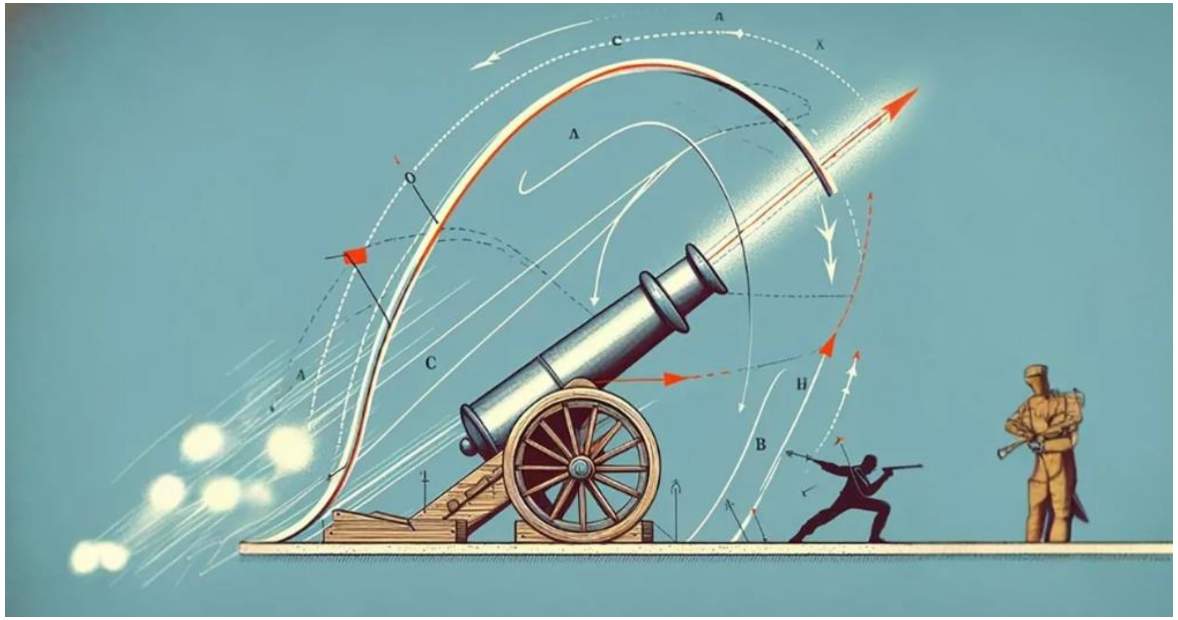
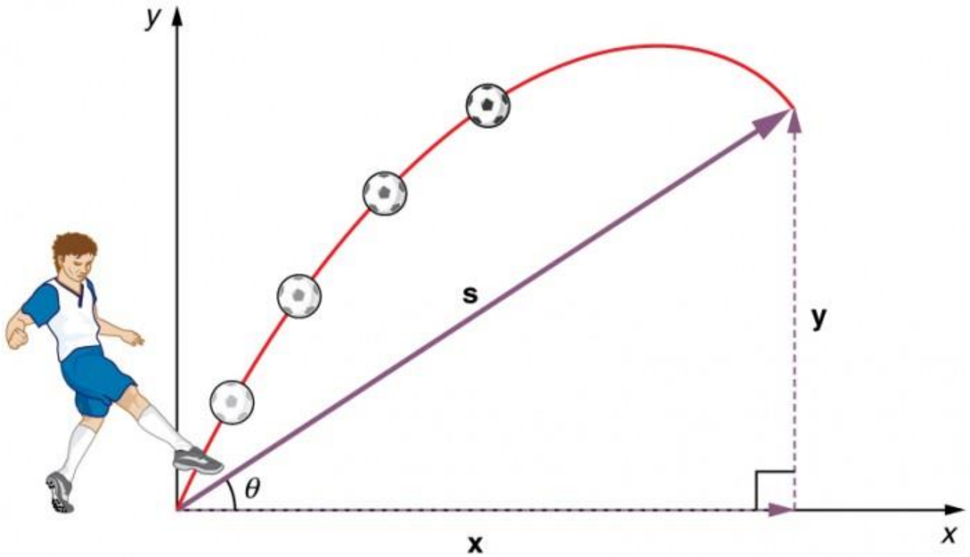




$$x(t) = V_0 t cos(\theta)$$

$$y(t) = -\frac{1}{2}gt^2 + V_0 t sin\theta$$

$$x = t^2 - t$$

$0 = t^2 - t \Rightarrow t(t-1) = 0 \begin{cases} t = 0 \\ t = 1 \end{cases}$

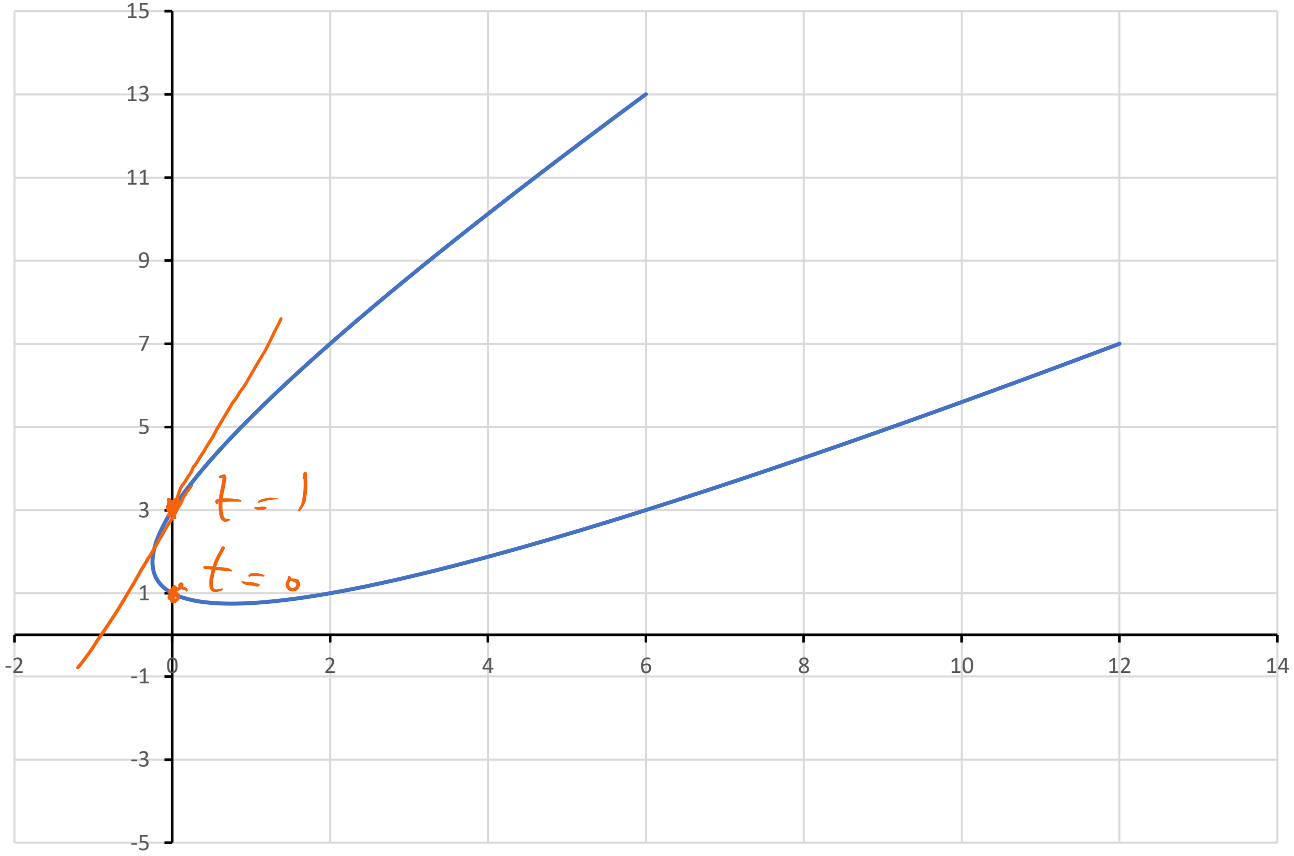
$y - y_0 = m(x - x_0)$

$$y = t^2 + t + 1$$

$$(x, y) = (0,3)$$

$m = \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2t+1}{2t-1} = \dfrac{3}{1} = 3$

$y = 3x + 3$



$x = t^2 - 2t$ $= 8$

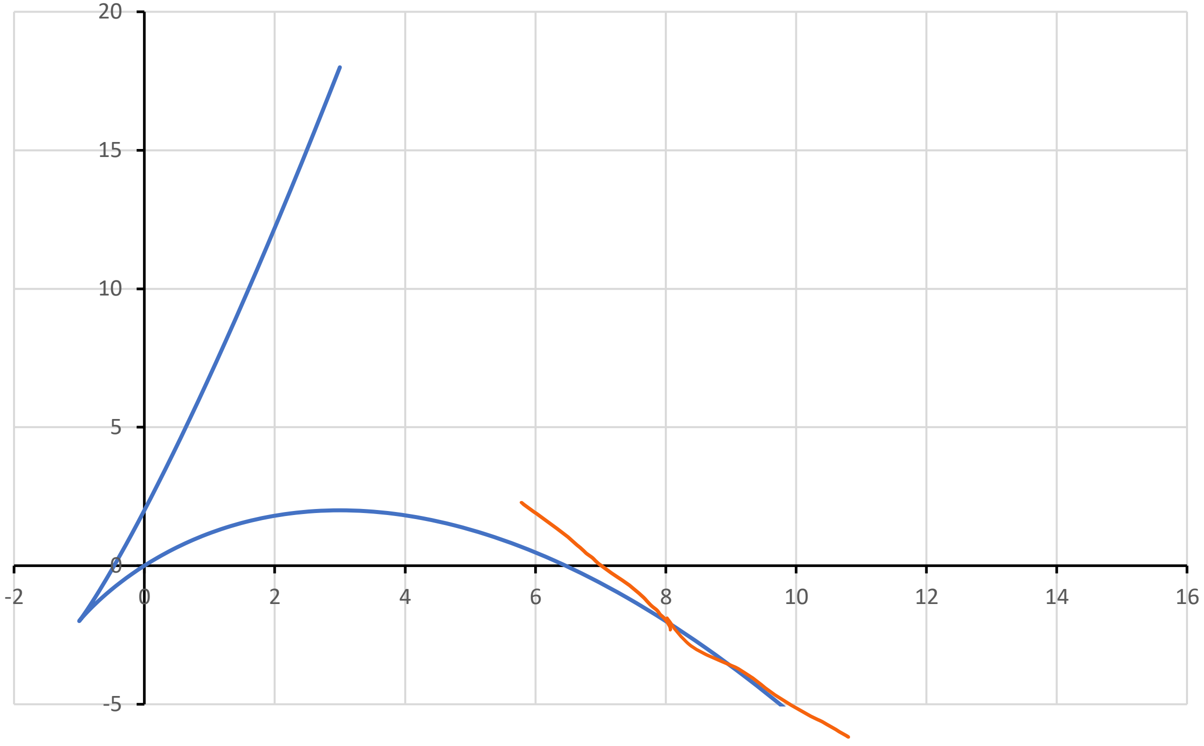
$y = t^3 - 3t$ $-8 + 6 = -2$

$t = -2$

$y - (-2) = m(x - 8)$

$$m = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 3}{2t - 2} = \frac{12 - 3}{-6} = \frac{+9}{-6} = -\frac{3}{2}$$

$$y = -\frac{3}{2}(x - 8) - 2 = -\frac{3}{2}x + 10$$

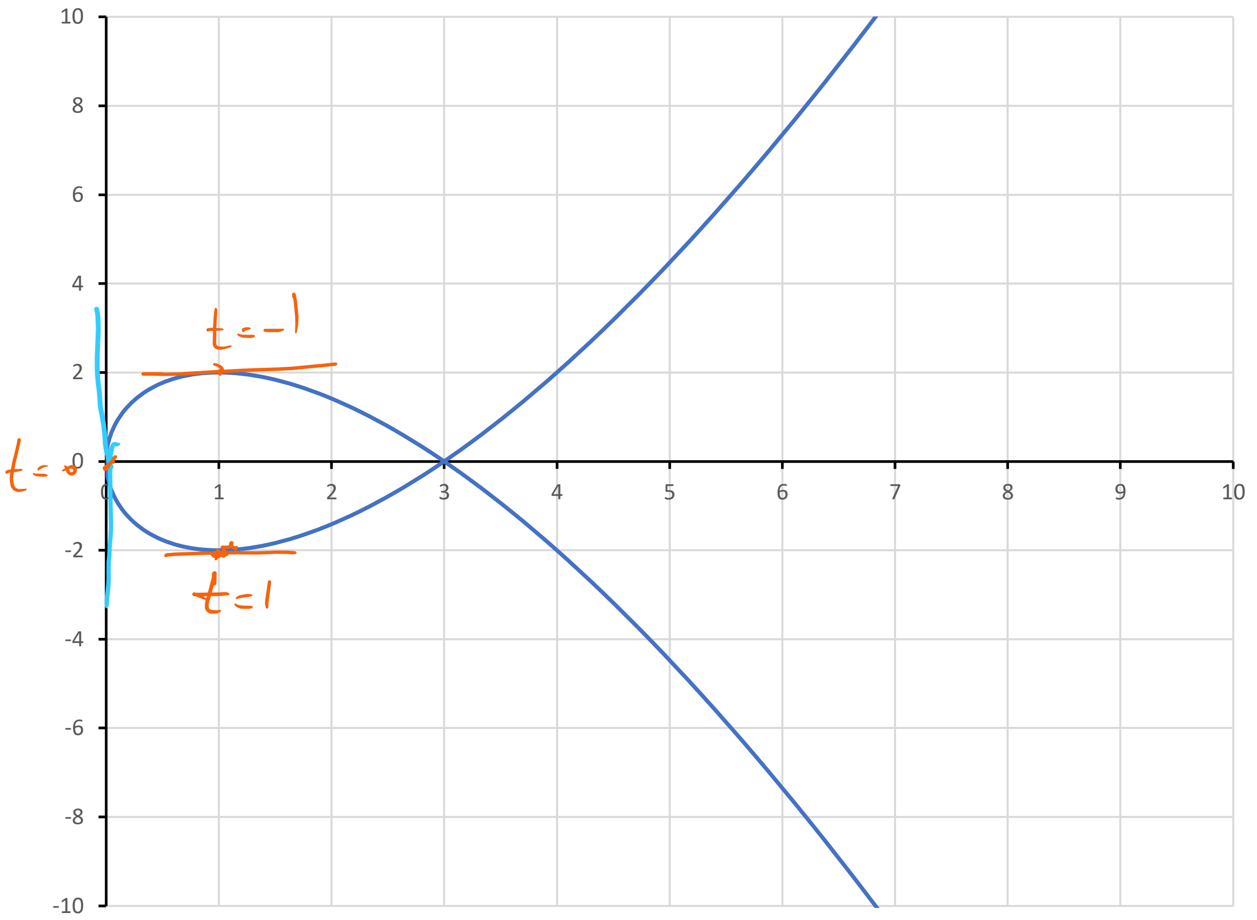


$x = t^2$

$y = t^3 - 3t$

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \begin{cases} \boxed{\frac{dx}{dt} = 0 \,,\ \frac{dy}{dt} \neq 0} \\ \\ \frac{dx}{dt} \neq 0 \,,\ \frac{dy}{dx} = 0 \end{cases}$$

مماس قائم



$$\frac{dy}{dt} = 3t^2 - 3 = 3(\underbrace{t^2 - 1}_{0}) \Rightarrow \begin{cases} t = -1 \\ t = 1 \end{cases}$$

$$\frac{dx}{dt} = 2t$$

$$y = t^3 - 3t \Rightarrow y = 0 \Rightarrow t(t^2 - 3) = 0 \begin{cases} t = 0 \\ t = \pm\sqrt{3} \checkmark \end{cases}$$

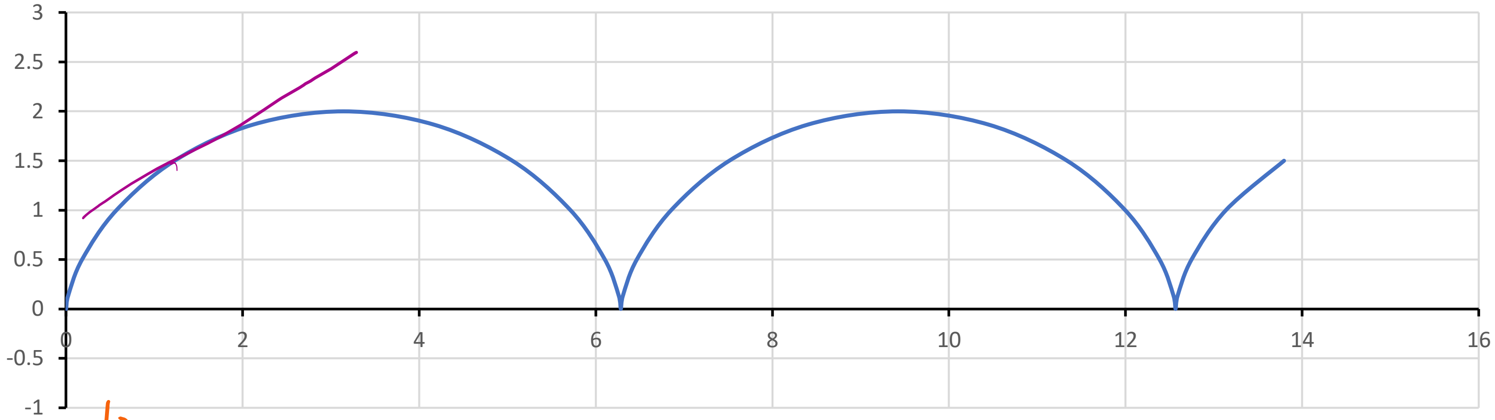
$$m = \frac{3t^2 - 3}{2t} \Rightarrow \frac{6}{2\sqrt{3}} = \sqrt{3} \,,\ -\sqrt{3}$$

$$t = \sqrt{3} \Rightarrow \underline{x = 3} \,,\ \underline{y = (\sqrt{3}^3 - 3\sqrt{3})}$$

$$x = r(\theta - \sin(\theta))$$

$$y = r(1 - \underbrace{\cos(\theta)}_{-\sin\theta})$$

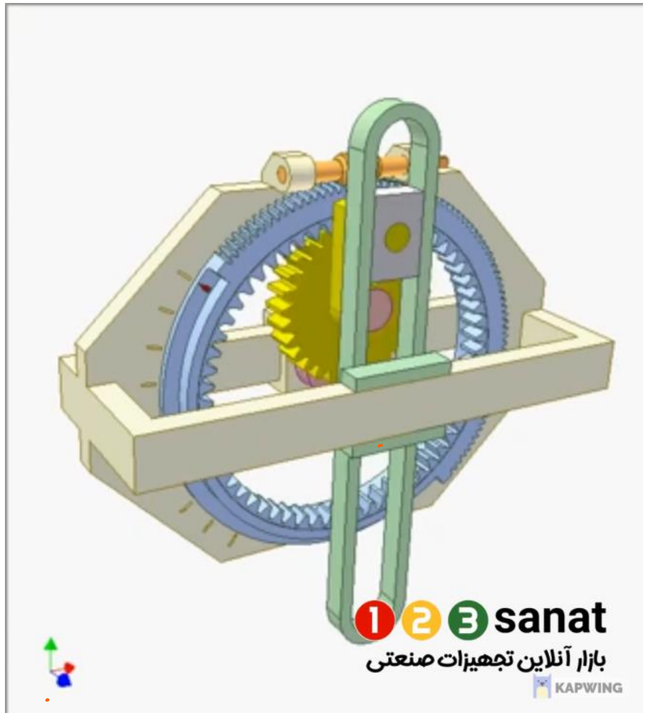
$$\theta = \frac{\pi}{3}$$



$$m = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cancel{r}(\sin\theta)}{\cancel{r}(1 - \cos\theta)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}, \quad x, y$$

$$x = \underbrace{r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)}, \quad y = r\left(\tfrac{1}{2}\right) \quad \frac{r}{2}$$

$$y - \frac{r}{2} = \sqrt{3}\left(x - r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right)$$

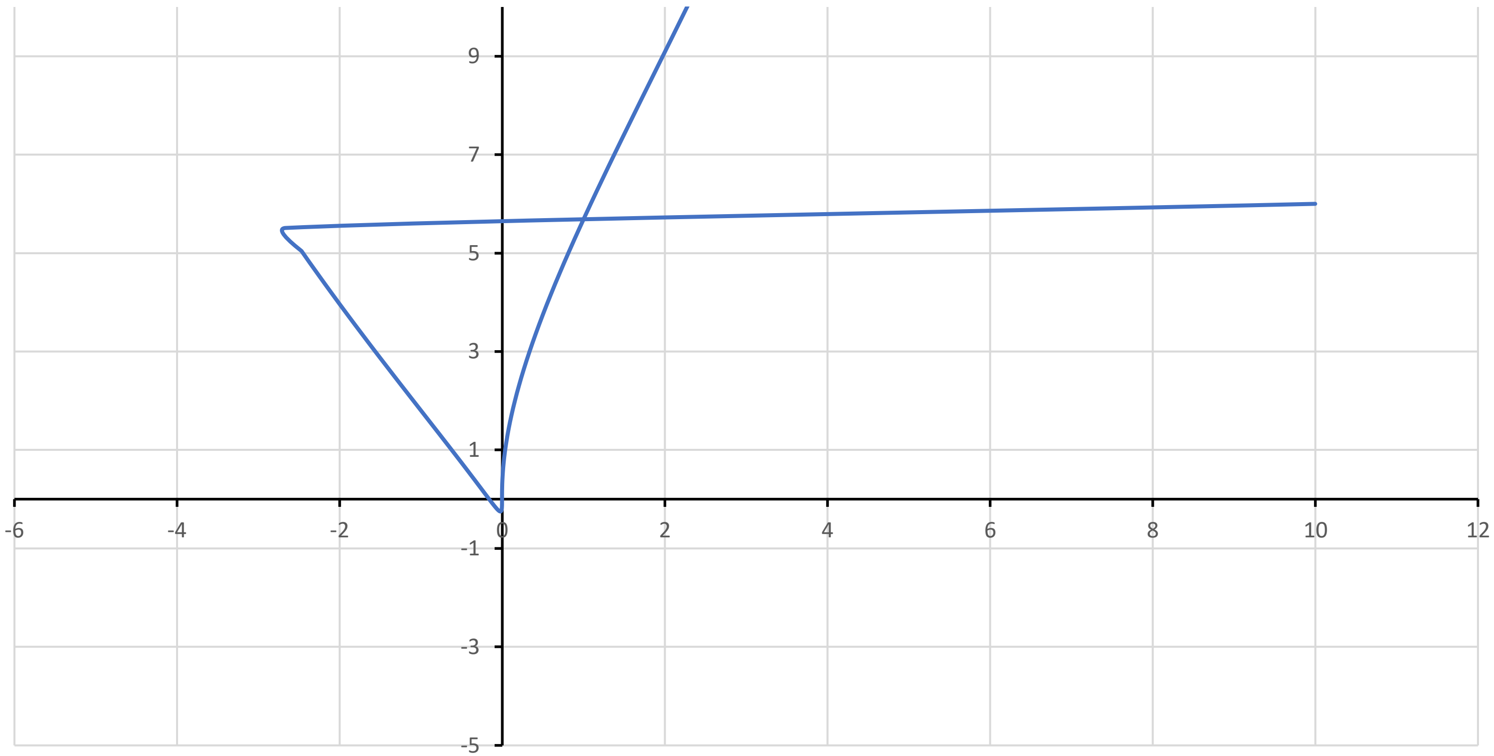


$$x = t^2 + 1$$

$$y = t^2 + t$$

$$\frac{d^2y}{dx^2} = \underbrace{\frac{d}{dx}\left(\frac{dy}{dx}\right)}$$

$$\underbrace{\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}} = \frac{2t+1}{2t}$$



$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{\overset{4t}{2(\cancel{2t})} - \overset{-4t}{2(2t+1)}}{(2t)^2}}{2t} = \frac{-2}{8t^3} = \frac{-1}{4t^3} \Rightarrow$$

تقعر

$$x = t^2$$

$$y = t^3 - 3t$$

$$\frac{d^2y}{dx^2} = ? \quad \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dt} = 3t^2 - 3 \qquad \frac{dx}{dt} = 2t$$

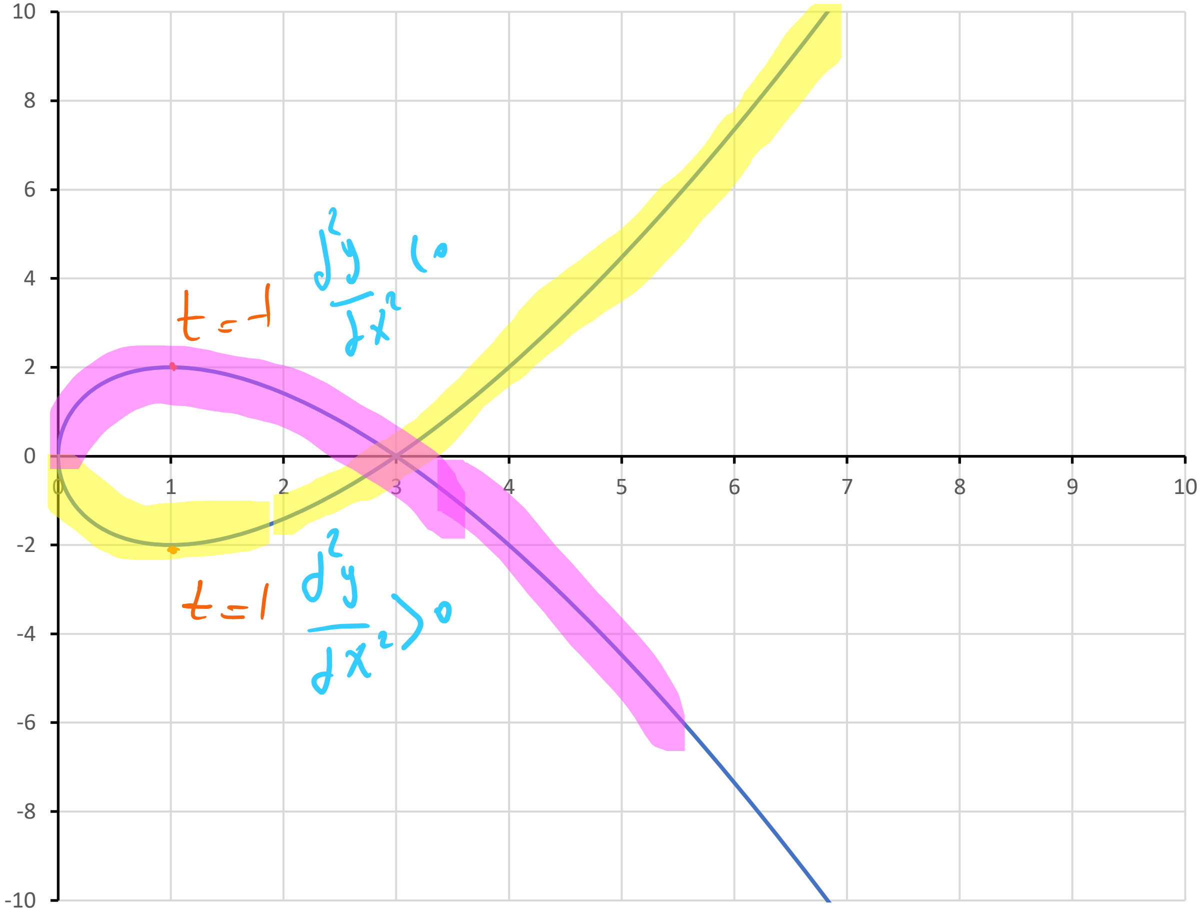
$$\frac{dy}{dx} = \frac{3t^2-3}{2t} \xrightarrow{\frac{d}{dt}} \frac{\overset{12t^2}{6t(2t)} - \overset{-6t^2+6}{\cancel{2(3t^2-3)}}}{4t^2}$$

$$= \frac{6t^2+6}{4t^2}$$

$$\frac{dx}{dt} = 2t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{6t^2+6}{4t^2}}{2t} = \frac{6(t^2+1)}{8t^3} = \frac{3(t^2+1)}{4t^3}$$

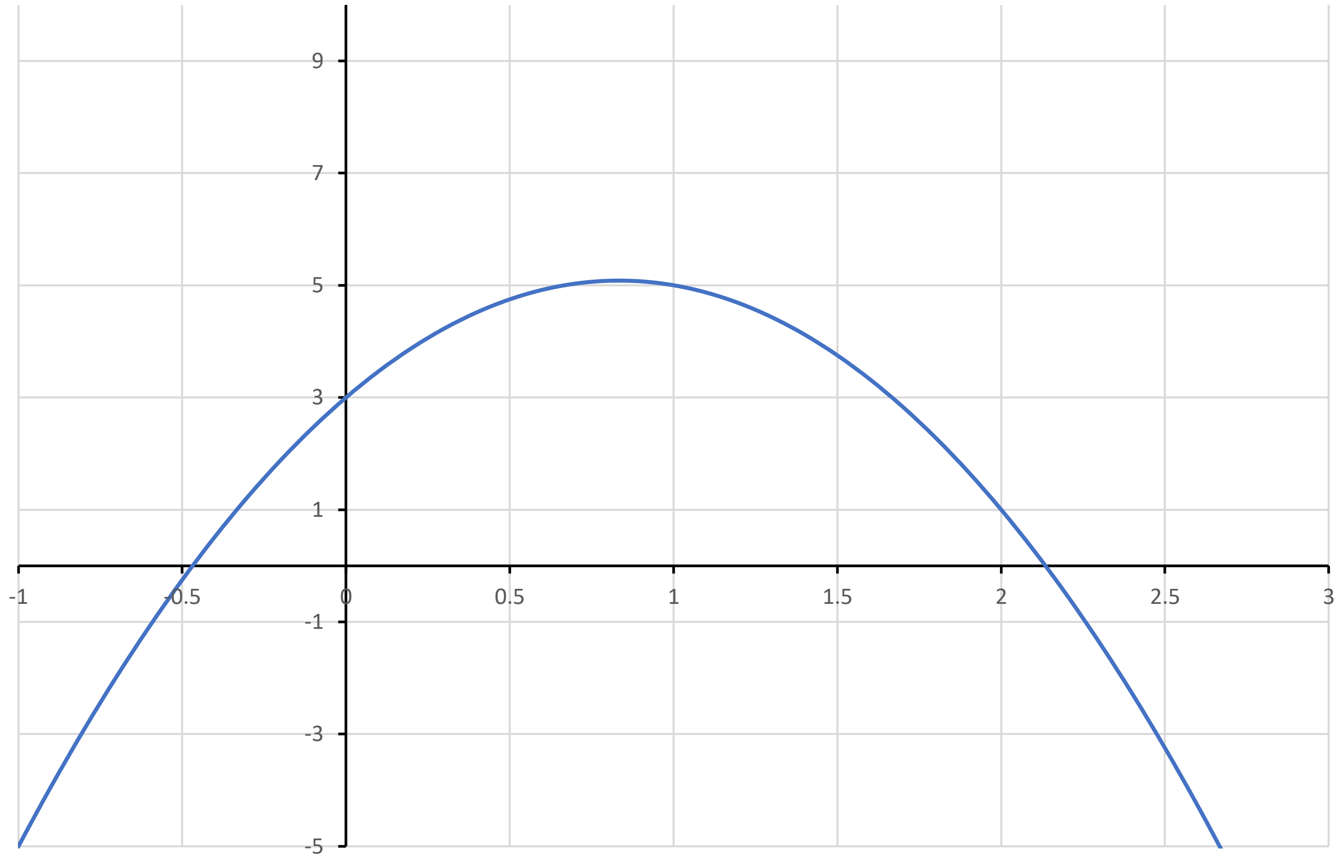
$+$



$$A = \int_a^b y\,dx = \int_\alpha^\beta g(t)f\acute{}(t)dt$$

$x = f(t)$ => $dx = f'(t)dt$

$y = g(t)$



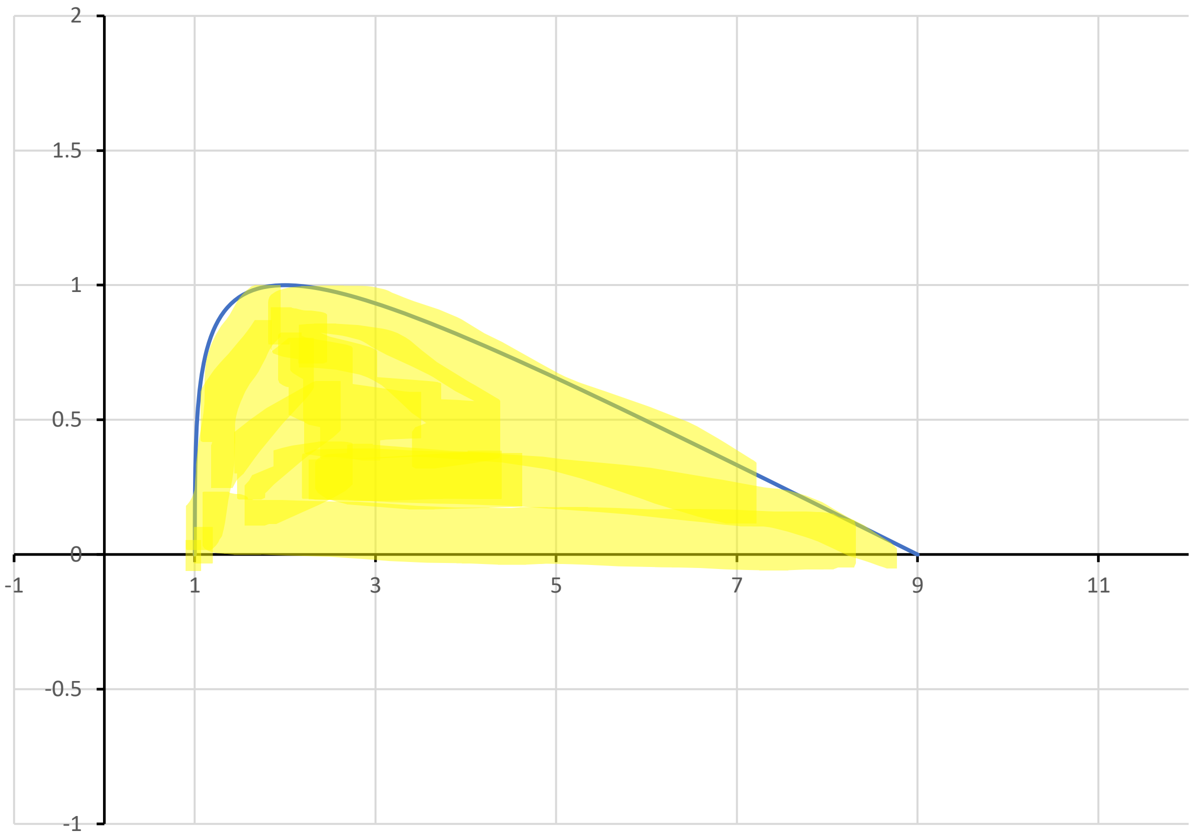
$$A = \int_a^b y\,dx = \int_\alpha^\beta \overline{g(t)} f'(t)\,dt$$

$x = t^3 + 1 \Rightarrow dx = 3t^2\,dt$ $\quad 0 < t < 2$

$y = 2t - t^2$

$$A = \int_0^2 (2t - t^2)\, 3t^2\, dt = 3\int_0^2 (2t^3 - t^4)\, dt$$

$$= 3\left[\frac{2t^4}{4} - \frac{t^5}{5}\right]_0^2 = \frac{24}{5}$$

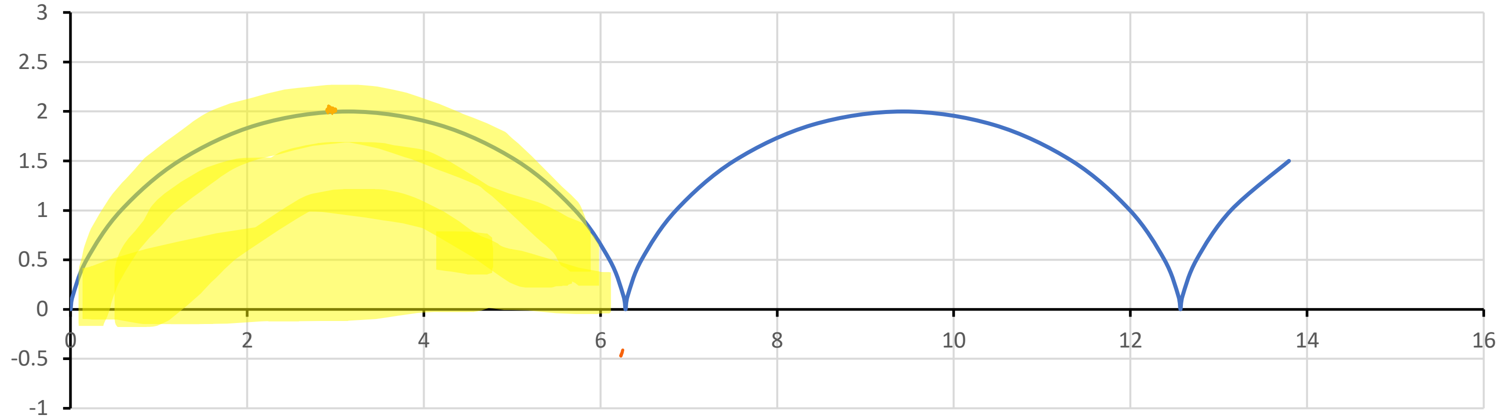


$$dx = r(1-\cos\theta)\,d\theta$$

$$x = r(\theta - \sin(\theta))$$

$$y = r(1-\cos(\theta))$$

$$0 < \theta < 2\pi$$



$$A = \int y\,dx = \int_0^{2\pi} g(\theta)\,f'(\theta)\,d\theta$$

$$A = \int r^2(1-\cos\theta)(1-\cos\theta)\,d\theta$$

$$A = r^2\int (1-\cos\theta)^2\,d\theta = r^2\int_0^{2\pi}\Big(1+\underbrace{\cos^2\theta}_{\frac{1}{2}(1+\cos 2\theta)}-2\cos\theta\Big)\,d\theta = r^2\left[\frac{3}{2}\theta+\frac{1}{2}\left(\frac{1}{2}\sin 2\theta\right)-2\sin\theta\right]_0^{2\pi}$$

$$\Downarrow$$

$$r^2\int_0^{2\pi}\left[\frac{3}{2}+\frac{1}{2}\cos 2\theta - 2\cos\theta\right]d\theta$$

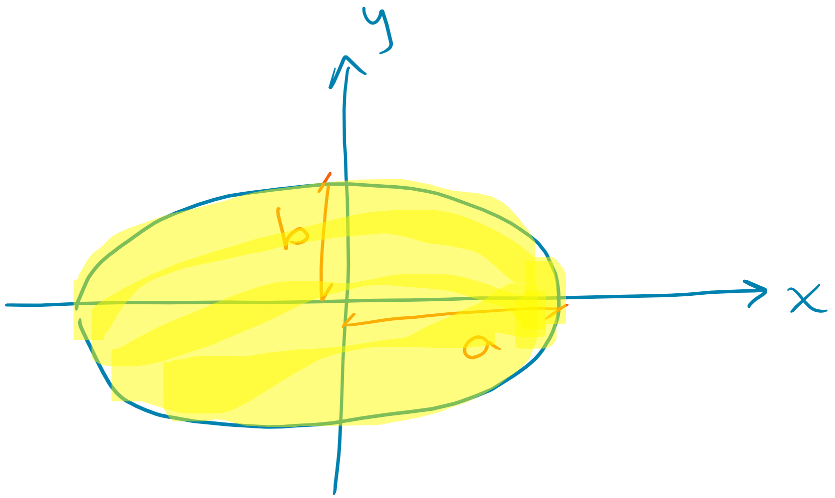
$$= r^2\left[\frac{3}{2}(2\pi)\right] = \underline{3\pi r^2}$$

$$x = a\cos\theta$$
$$y = b\sin\theta$$

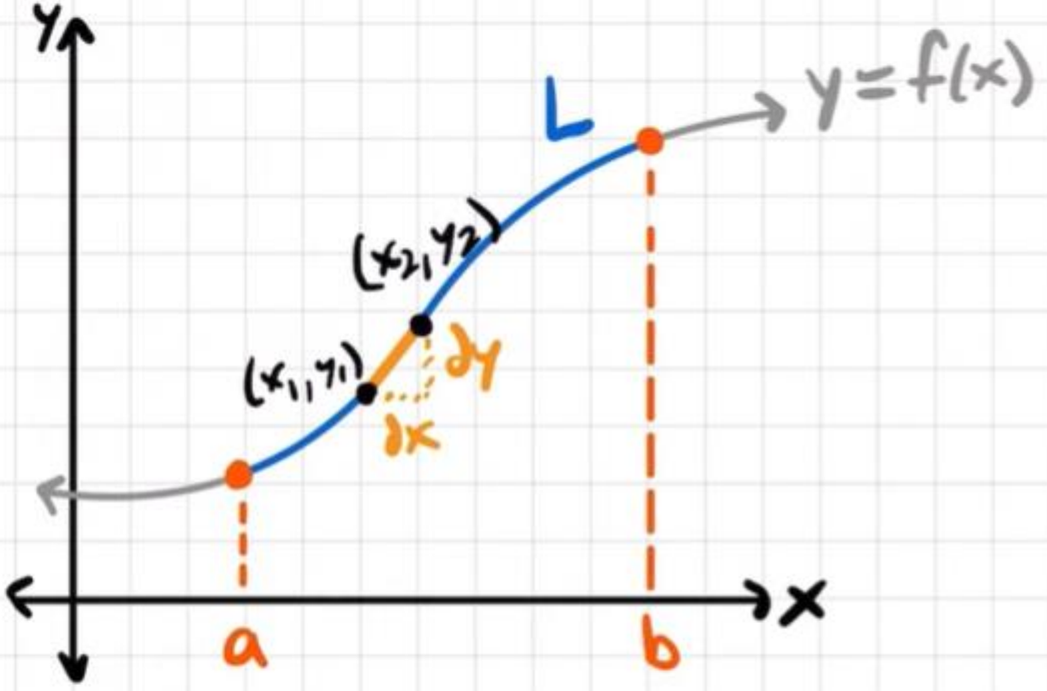
$$\left.\begin{aligned} x &= a\cos\theta \\ y &= b\sin\theta \end{aligned}\right\} \quad \frac{x}{a} = \cos\theta \qquad \frac{y}{b} = \sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Lesson 6: Arc Length



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$L = \int_a^b \sqrt{(\partial x)^2 + (\partial y)^2}$$

$$= \int_a^b \sqrt{(\partial x)^2 \left(1 + \frac{(\partial y)^2}{(\partial x)^2}\right)}$$

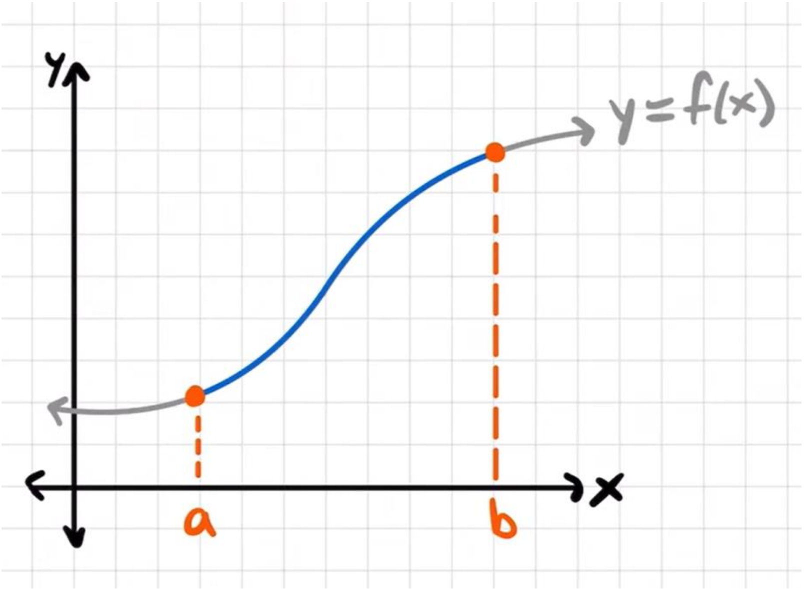
$$= \int_a^b \partial x \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}$$

$$= \int_a^b \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2}\, \partial x$$

$$\boxed{= \int_a^b \sqrt{1 + (f'(x))^2}\, \partial x}$$

https://www.youtube.com/watch?v=dAXOz8ykrjQ

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

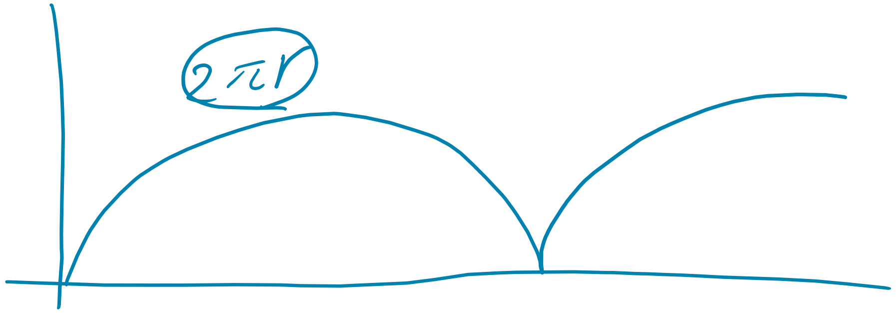


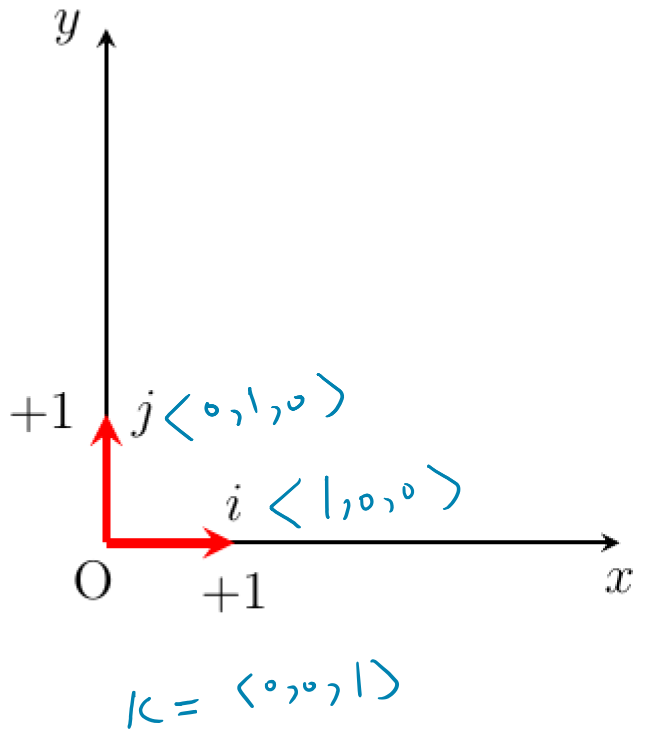
$$x = f(t)$$
$$y = g(t)$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy/dt}{dx/dt}\right)^2} \times \frac{dx}{dt}\, dt = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

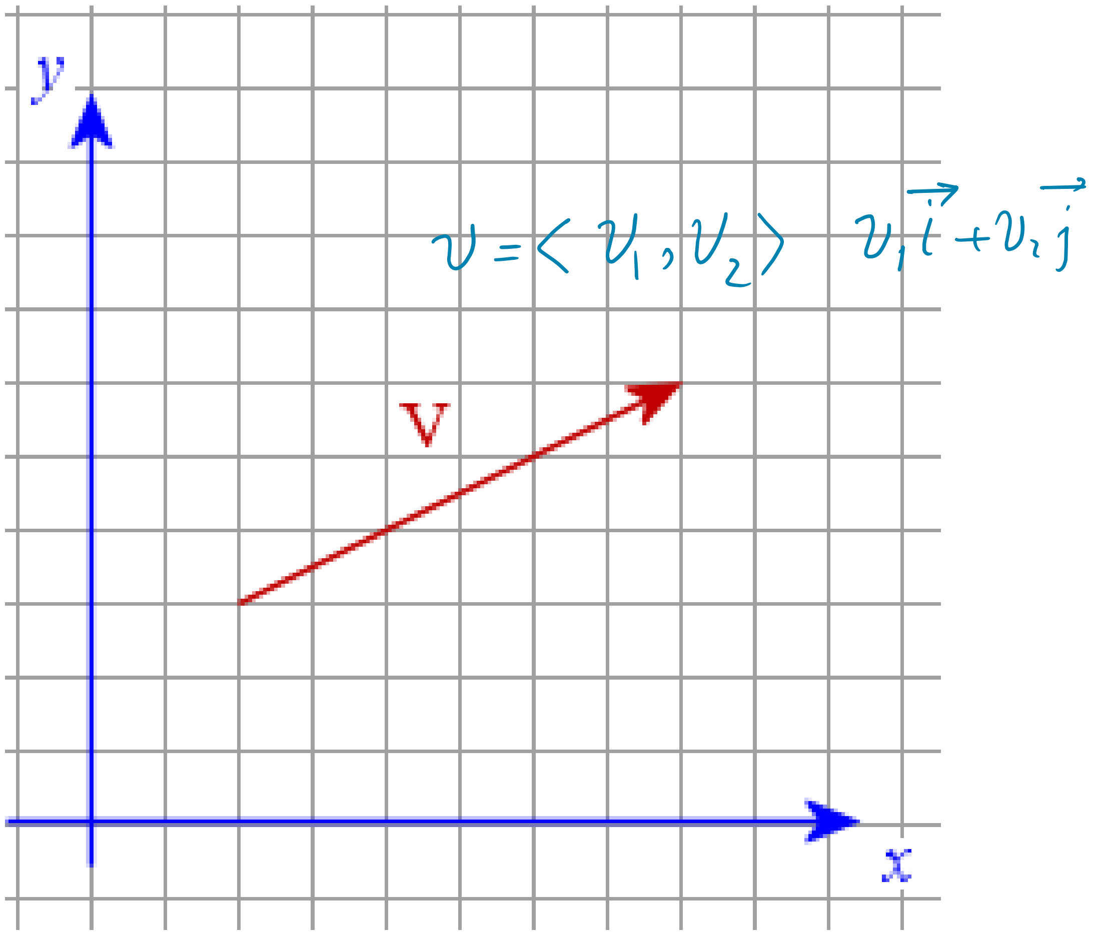
$$\begin{cases} x = r(\theta - \sin\theta) \Rightarrow \dfrac{dx}{d\theta} \\ y = r(1 - \cos\theta) \Rightarrow \dfrac{dy}{d\theta} \end{cases}$$

$\Rightarrow$ طول کمان

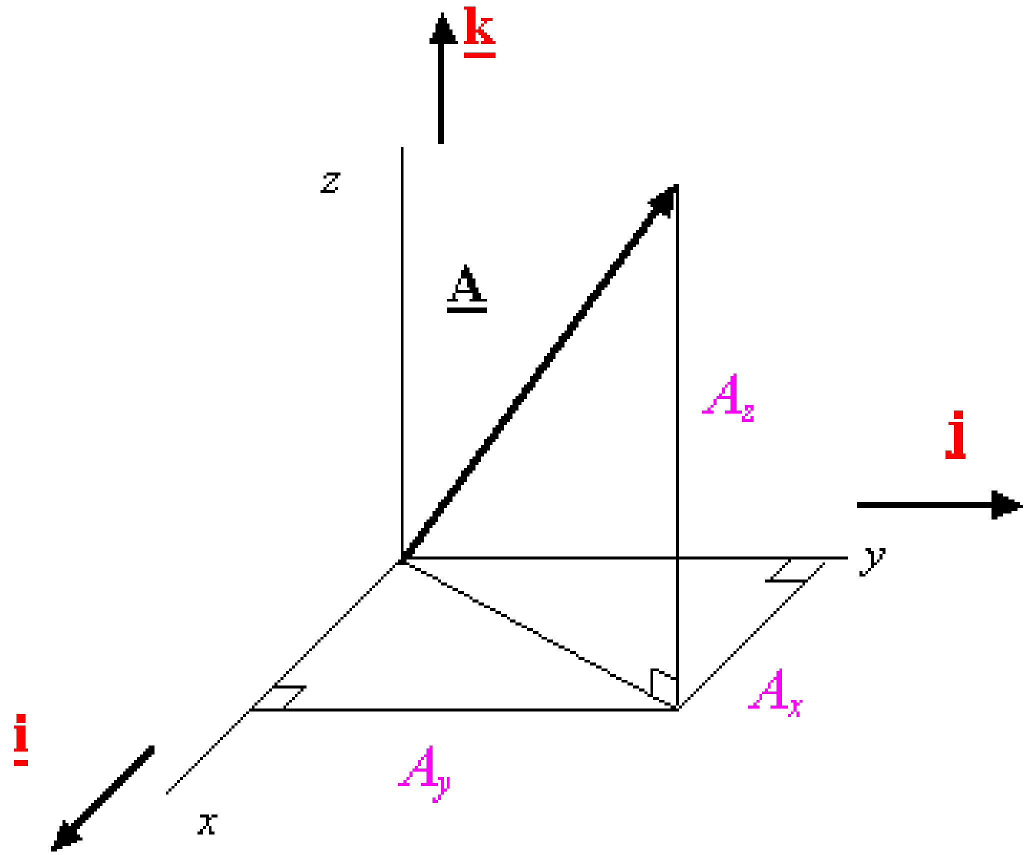




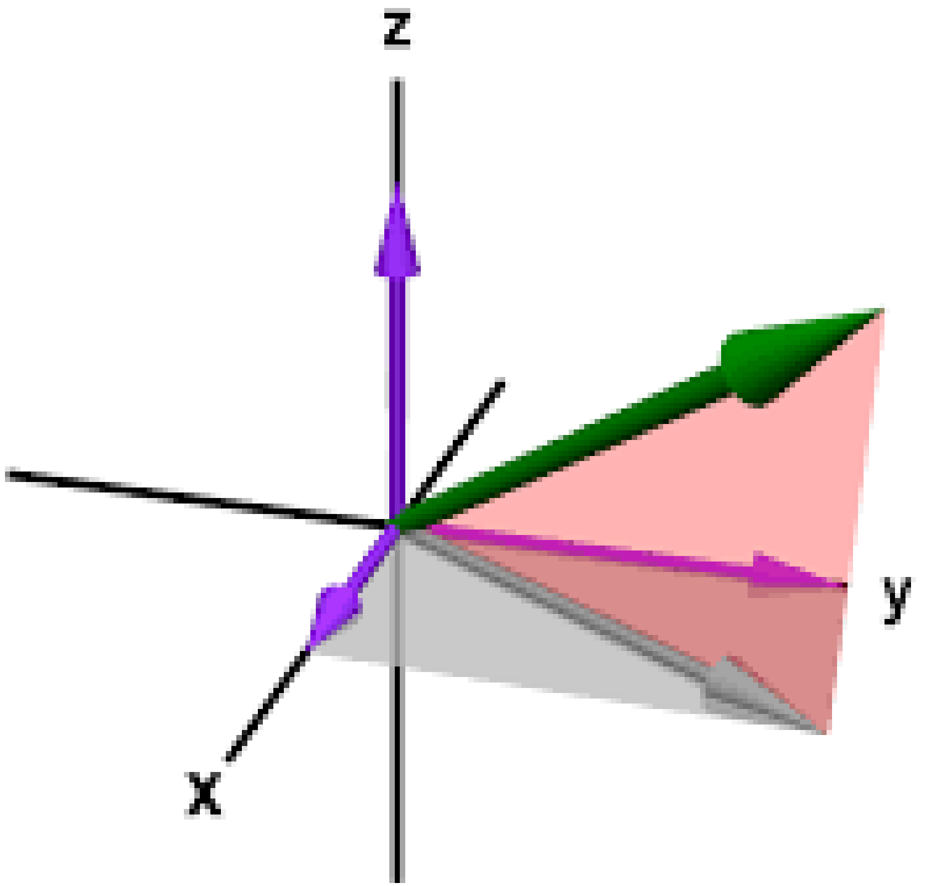
y
+1
j <0,1,0>
i <1,0,0>
O
+1
x
k = <0,0,1>



y
v = <v1, v2>
v1 i + v2 j
v
x



k
z
A
A_z
i
y
A_x
i
A_y
x



z
y
x

مثال: بردار واحد هم جهت با بردار <۶ و ۲-> را پیدا کنید.

$$v = \langle v_1, v_2, v_3 \rangle$$

$$|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$\frac{v}{|v|} \Rightarrow$$ بردار واحد

$$v = 6\vec{i} - 2\vec{j}$$

$$\left\langle \frac{v_1}{|v|}, \frac{v_2}{|v|} \right\rangle = \left\langle \frac{6}{2\sqrt{10}}, \frac{-2}{2\sqrt{10}} \right\rangle$$

$$|v| = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

$a = \langle a_1, a_2 \rangle$

$b = \langle b_1, b_2 \rangle$

$a + b = \langle a_1 + b_1, a_2 + b_2 \rangle$

$a + b = b + a$ خاصیت جابجایی دارد

$a + (b + c) = (a + b) + c$

$a + 0 = a$

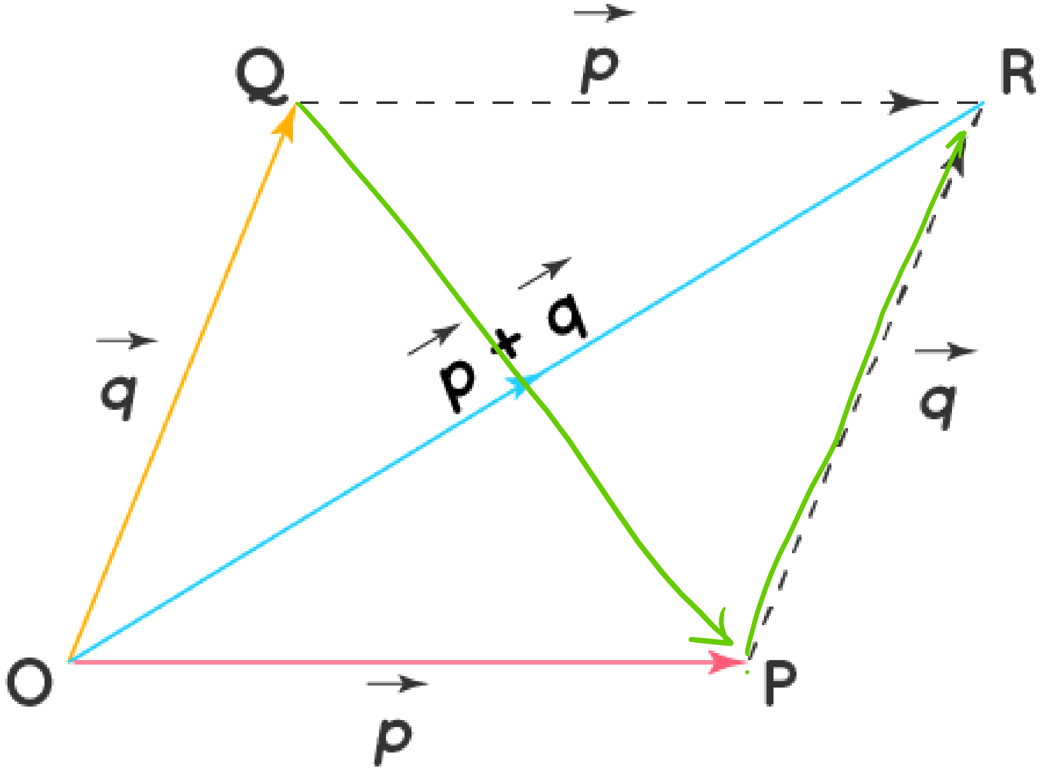
$a + (-a) = 0$

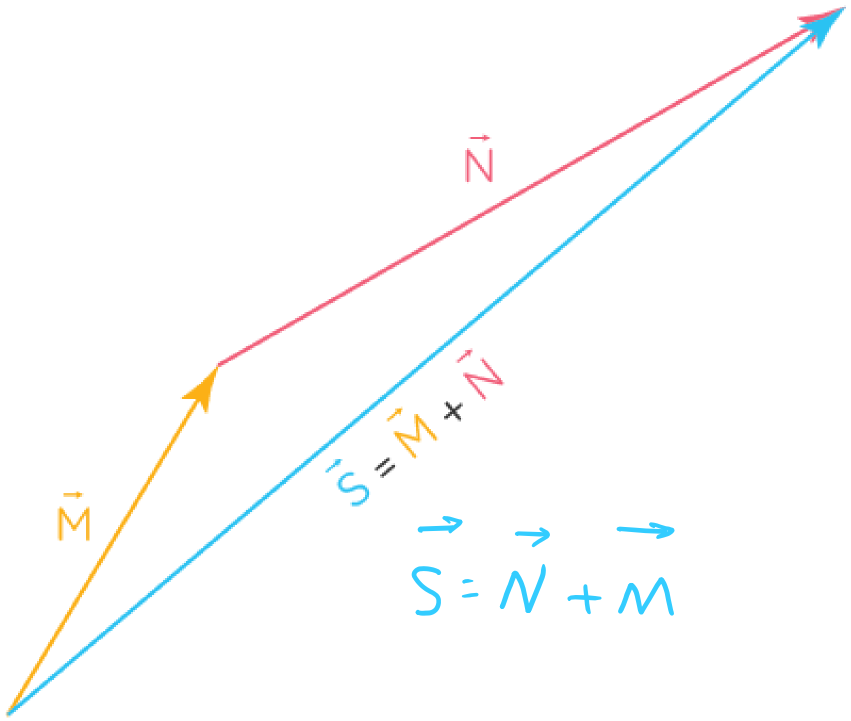
$c(a + b) = ca + cb$

$(cd)a = c(da)$

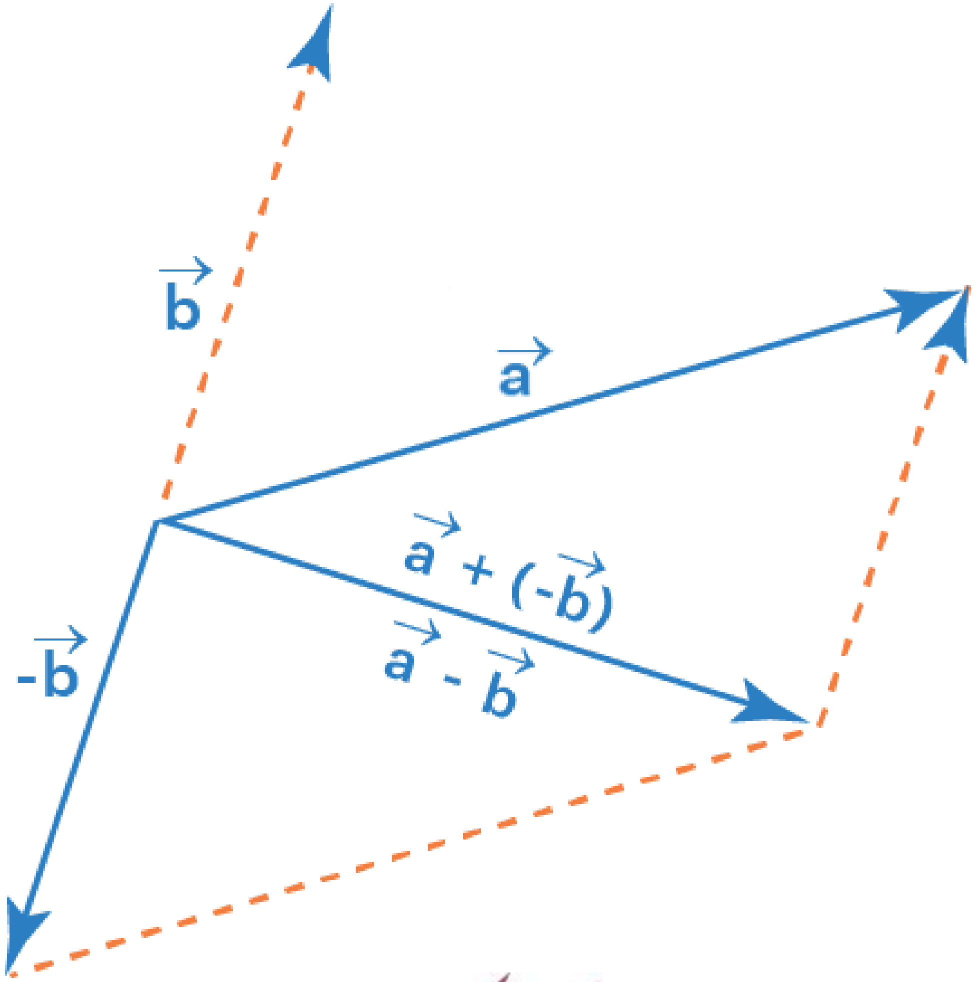
$1a = a$

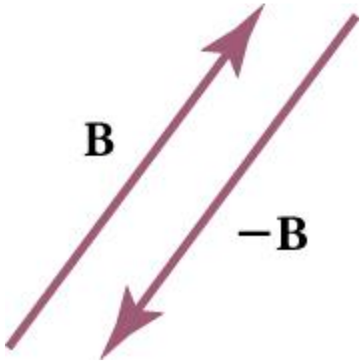
بردار $\begin{cases} \text{جهت} \\ \text{طول} \end{cases}$ $\Rightarrow$ $\vec{p} + \vec{q} = \vec{q} + \vec{p}$ خاصیت جابجایی

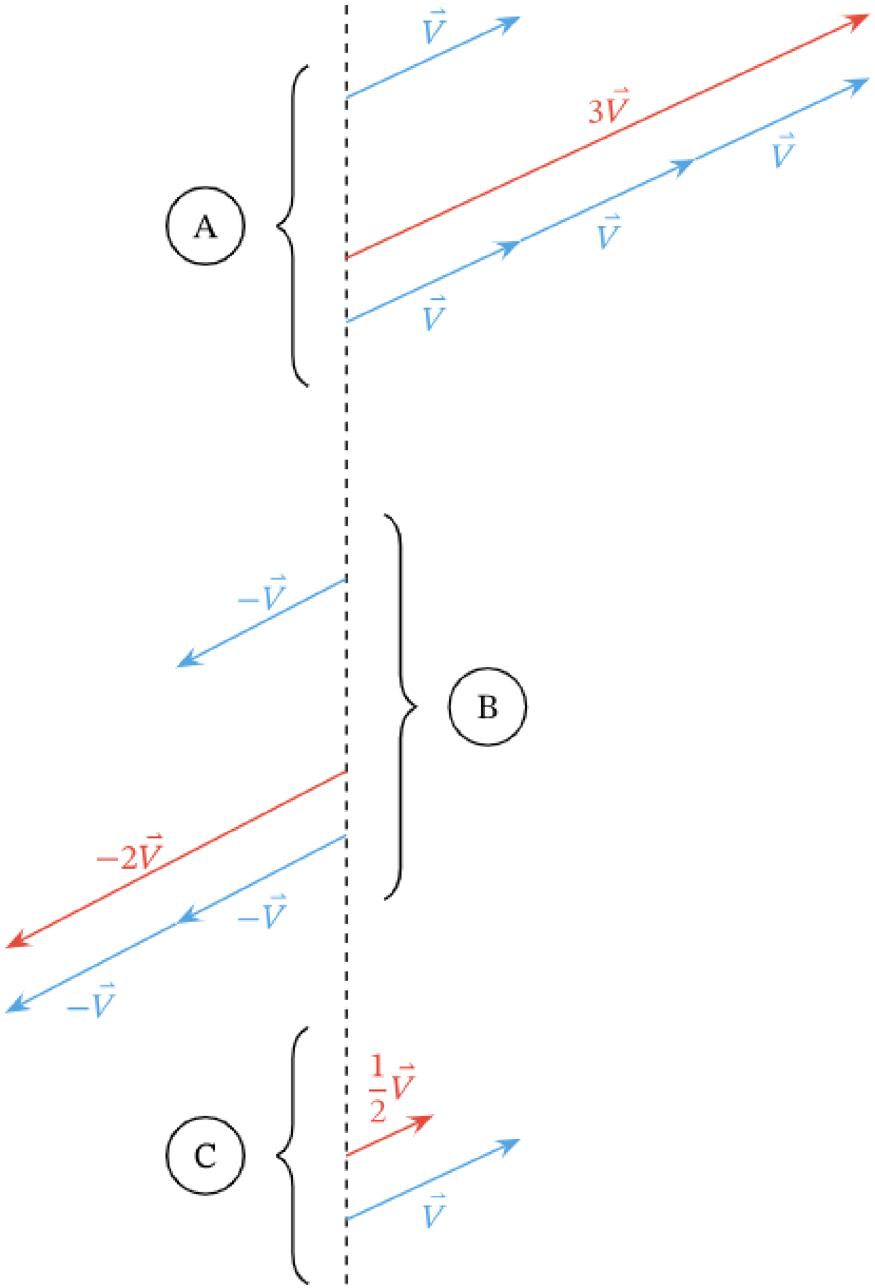




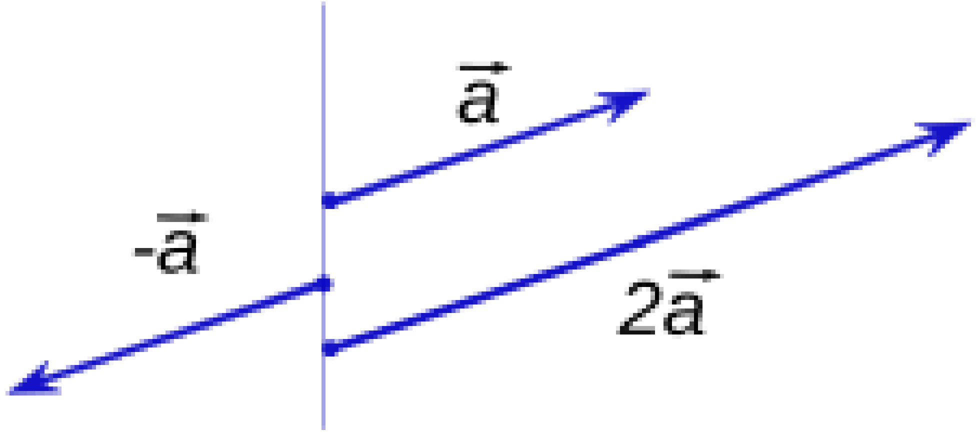
$a - b = a + (-b)$







A
B
C
$\vec{V}$
$3\vec{V}$
$-\vec{V}$
$-2\vec{V}$
$\frac{1}{2}\vec{V}$



$\vec{a}$
$-\vec{a}$
$2\vec{a}$

Dot product ضرب نقطه‌ای

$a = \langle a_1, a_2, \dots, a_n \rangle$

$b = \langle b_1, b_2, \dots, b_n \rangle$

$a \cdot b = \underbrace{a_1 b_1 + a_2 b_2 + \dots + a_n b_n}_{\text{یک عدد}}$

$a \cdot a = |a|^2$

$a \cdot b = b \cdot a$ جابجایی

$a \cdot (b + c) = a \cdot b + a \cdot c$

$a \cdot b = |a||b| \cos\theta$

$\left.\begin{array}{l} a = \langle 2, -1, 2 \rangle \\ b = \langle 1, -1, 0 \rangle \end{array}\right\}$ زاویه بین دو بردار $= ?$

$|a| = \sqrt{9} = 3$

$|b| = \sqrt{2}$

$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{\cancel{2+1}^{\,3} + 0}{\sqrt{2} \times 3} = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$ rd $45°$

$a: 2\vec{i} + 2\vec{j} - \vec{k}$

$b: 5\vec{i} - 4\vec{j} + 2\vec{k}$

$a \cdot b = 0 \Rightarrow 10 - 8 - 2 = 0$

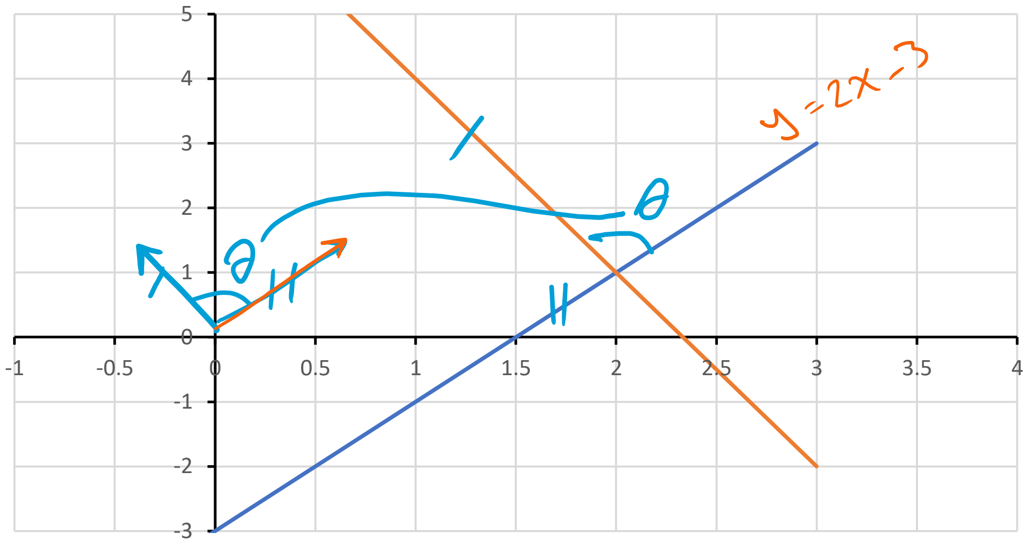
برهم عمودند

$$\begin{cases} 2x - y = 3 \\ y = 2x - 3 \\ 3x + y = 7 \end{cases}$$

$y = 7 - 3x$

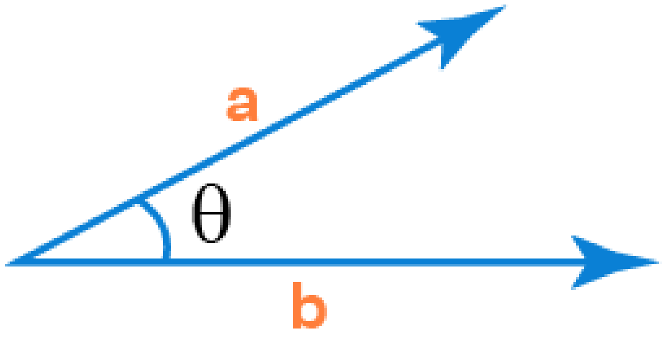
$a: \langle 1, 2 \rangle$

$b: \langle 1, -3 \rangle$



$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{1 - 6}{\sqrt{5} \times \sqrt{10}} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2} \Rightarrow \begin{matrix} \theta = 45^\circ \\ \theta = 135^\circ \end{matrix}$$

ویژگی‌های بردارها



$$\cos\theta = \frac{a.b}{|a|\ |b|}$$

(OR)

$$\sin\theta = \frac{|a \times b|}{|a|\ |b|}$$

https://www.cuemath.com/geometry/angle-between-vectors/

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

ماتریس همانی $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ماتریس قطری $= \begin{bmatrix} a_{11} & 0 & 6 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$

جمع و تفریق دو ماتریس

درایه به درایه

$$\begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 11 \\ 1 & 5 \end{bmatrix}$$

ضرب عدد در ماتریس

$$2 \times \begin{bmatrix} 3 & 2 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 2 & -18 \end{bmatrix}$$

$$\underbrace{A}_{n\times k} \times \underbrace{B}_{k\times m} = \underbrace{C}_{n\times m}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} \begin{bmatrix} 10 & 11 \\ 20 & 21 \\ 30 & 31 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 1\times 10 + 2\times 20 + 3\times 30 & 1\times 11 + 2\times 21 + 3\times 31 \\ 4\times 10 + 5\times 20 + 6\times 30 & 4\times 11 + 5\times 21 + 6\times 31 \end{bmatrix}_{2\times 2}$$

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 4 & 6 \\ 6 & 8 & 7 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 4 & 8 \\ 5 & 6 & 7 \end{bmatrix}$$

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$(A \times B)^T = B^T \times A^T \neq A^T \times B^T$$

* ماتریس متقارن

$$A = A^T$$

* اثر ماتریس

جمع مقادیر قطر اصلی $\Leftarrow tr(A) = \sum_{i=1}^{n} a_{ii}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad tr(A) = 1+5+9 = 15$$

* نرم ماتریس

$$\|A\|_F = \sqrt{\sum a_{ij}^2} = \sqrt{Tr(AA^T)}$$

* دترمینان ماتریس

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow det(A) = |A| = ad - bc$$

$$det\begin{bmatrix} 8 & 3 \\ 4 & 2 \end{bmatrix} = \underbrace{8\times 2}_{16} - \underbrace{3\times 4}_{12} = 4$$

$$A = \begin{bmatrix} a_+ & b_- & c_+ \\ d_- & e_+ & f_- \\ g_+ & h_- & i_+ \end{bmatrix}$$

$$a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$sgn = (-1)^{i+j}$$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

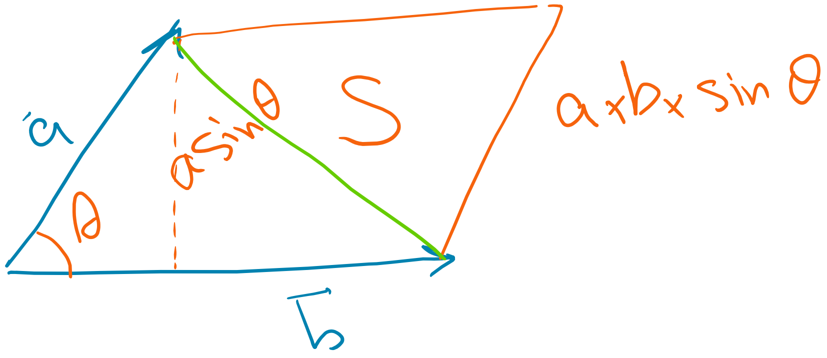
$$|A| = aei + bfg + cdh - ceg - afh - bdi$$

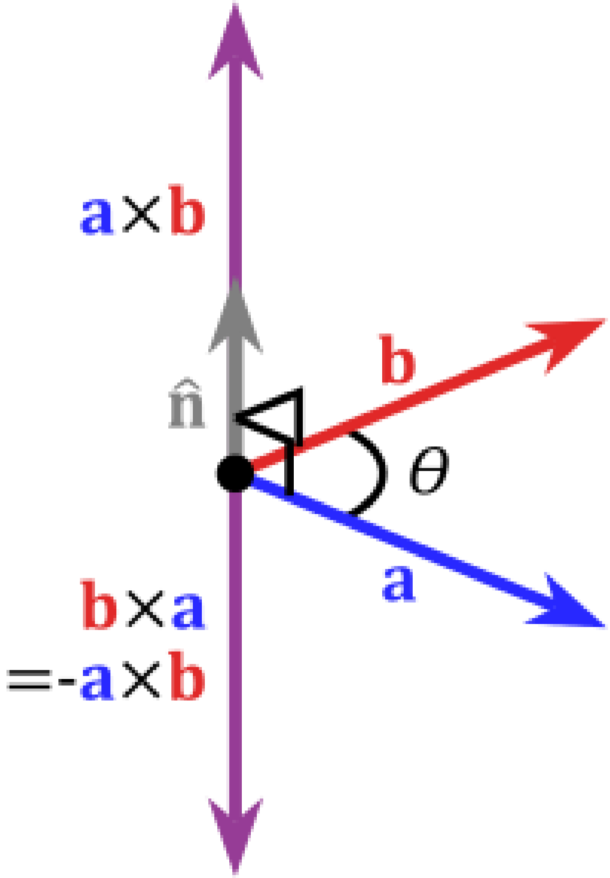
$a = \langle a_1, a_2, a_3 \rangle \quad a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$b = \langle b_1, b_2, b_3 \rangle \quad b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$|a \times b| = |a||b| \sin\theta$$





$A(0,0,0)$

$B(1,0,2)$ $\Rightarrow$ مساحت مثلث ؟

$C(2,-2,0)$