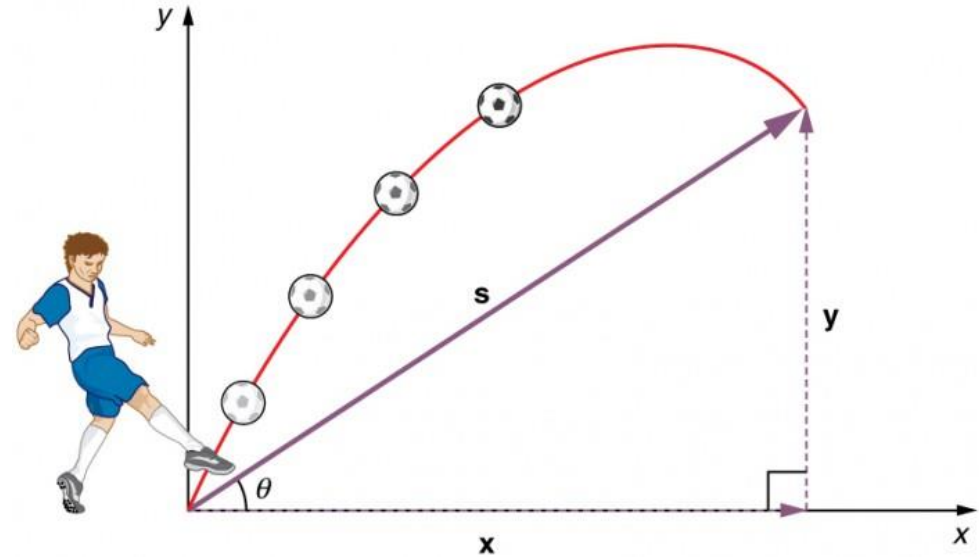
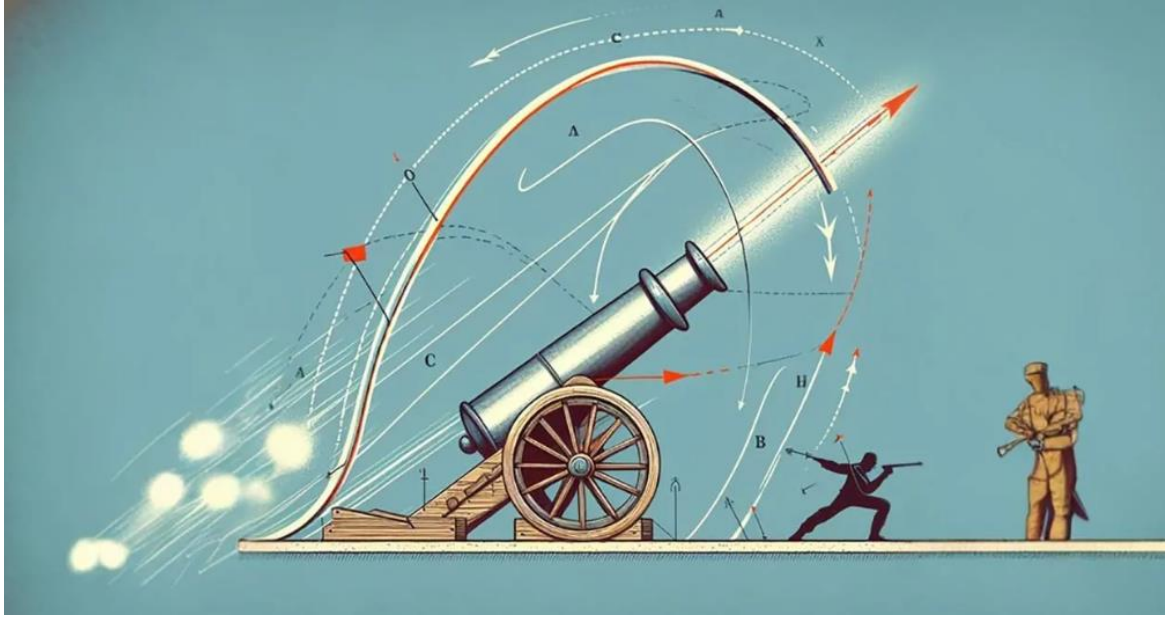




$$x(t) = V_0 t \cos(\theta)$$

$$y(t) = -\frac{1}{2}gt^2 + V_0 t \sin\theta$$

$$x = t^2 - t \quad \rightarrow \quad 0 = t^2 - t \Rightarrow t(t-1) = 0 \quad \left\{ \begin{array}{l} t=0 \\ t=1 \end{array} \right.$$

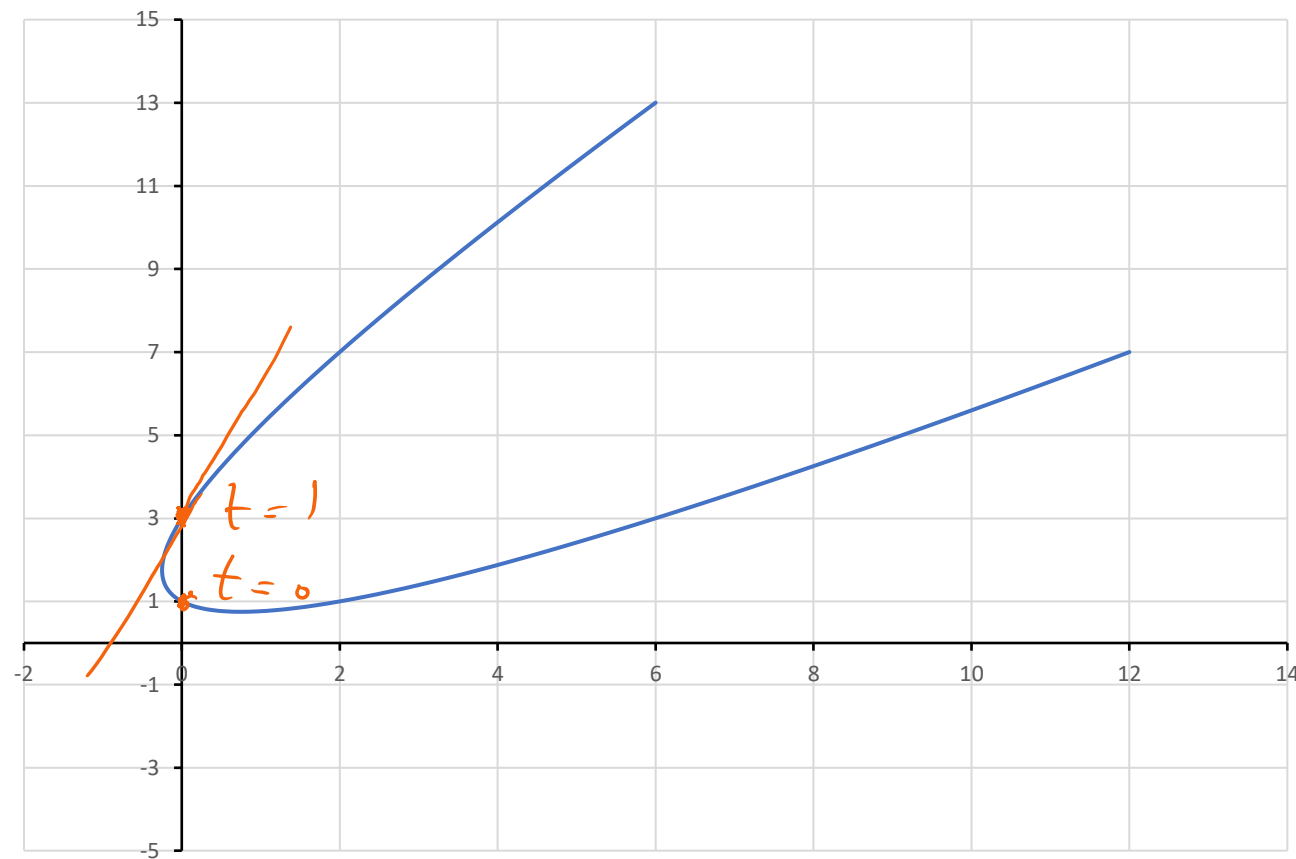
$$y - y_0 = m(x - x_0)$$

$$y = t^2 + t + 1$$

$$(x, y) = (0, 3)$$

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{2t-1} = \frac{3}{1} = 3$$

$$\underline{y = 3x + 3}$$



$$x = t^2 - 2t = 8$$

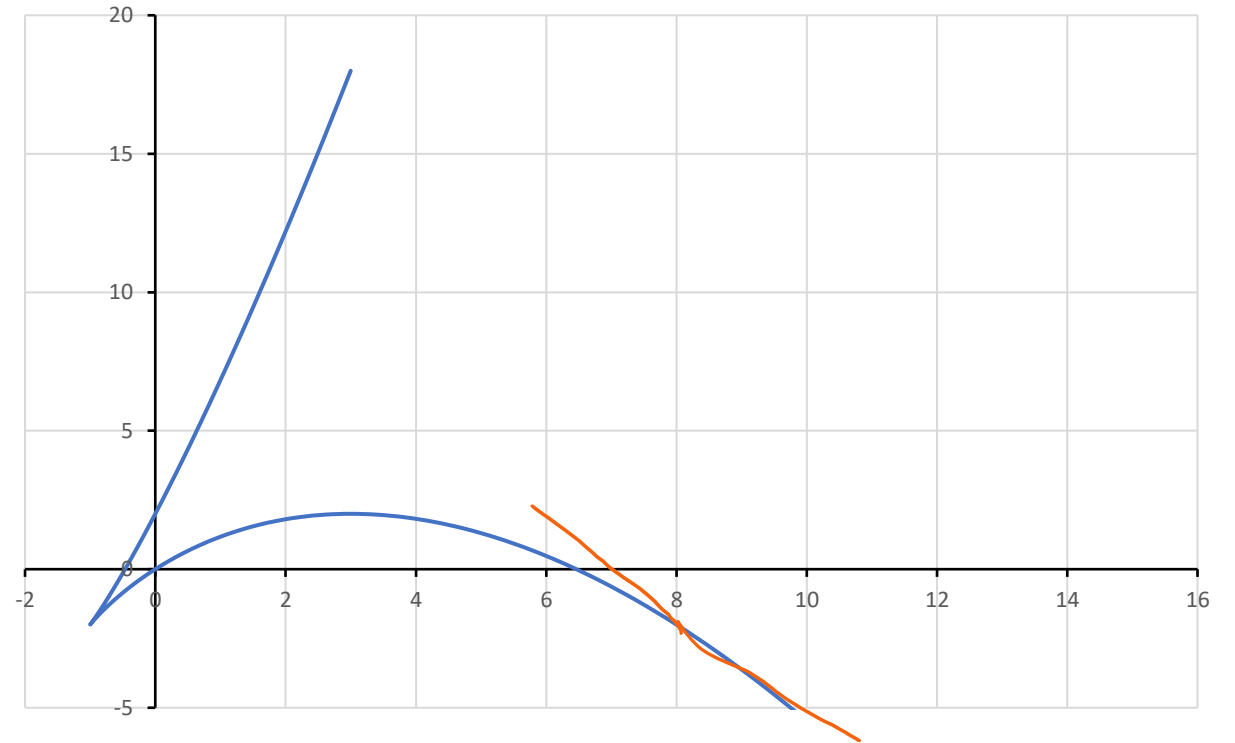
$$y - (-2) = m(x - 8)$$

$$y = t^3 - 3t \quad -8 + 6 = -2$$

$$t = -2$$

$$m = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 3}{2t - 2} = \frac{12 - 3}{-6} = \frac{+9}{-6} = -\frac{3}{2}$$

$$y = -\frac{3}{2}(x - 8) - 2 = -\frac{3}{2}x + 10$$



$$x = t^2$$

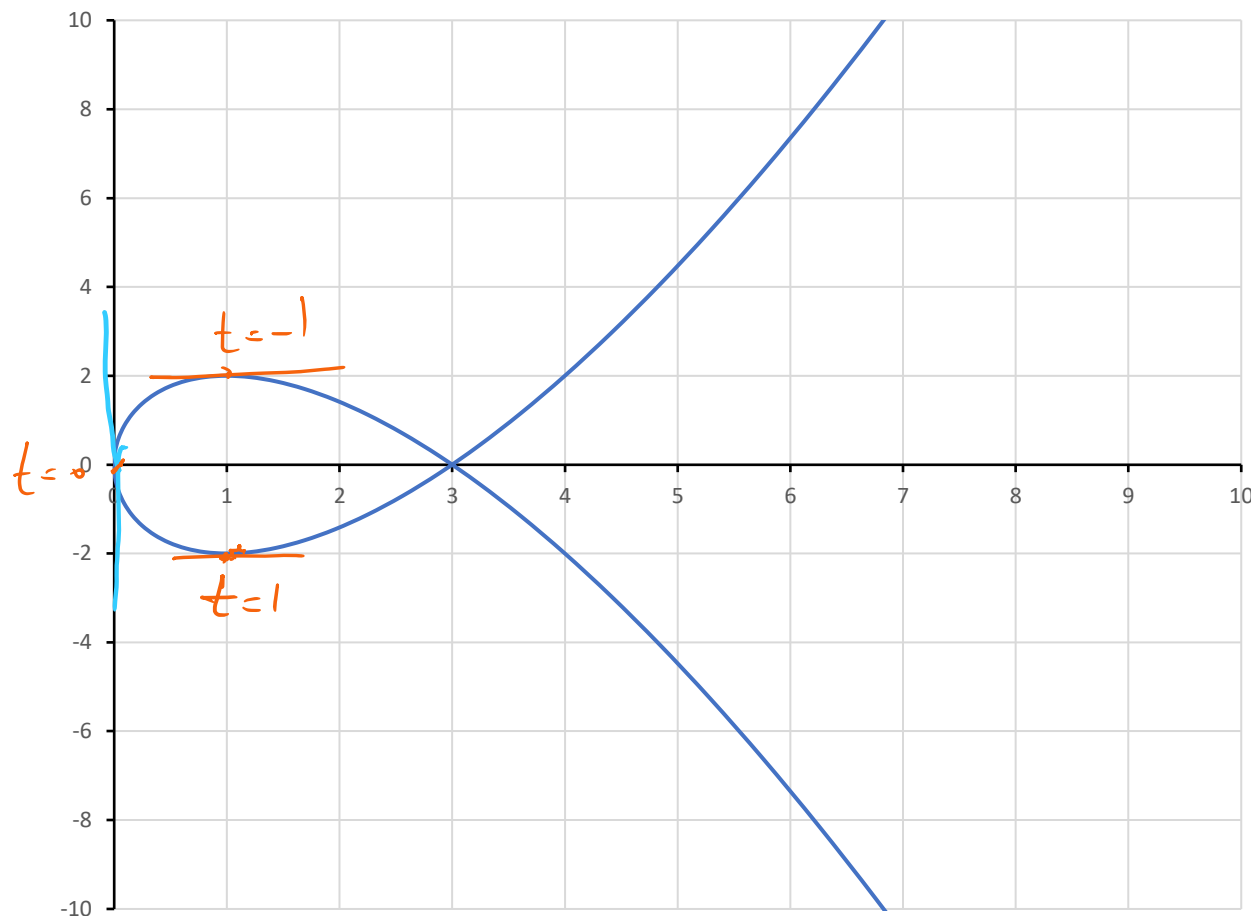
$$y = t^3 - 3t$$

نقاط مفردة

$$\frac{dx}{dt} = 0, \frac{dy}{dt} \neq 0$$

$$m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dx}{dt} \neq 0, \frac{dy}{dx} = 0$$



$$\frac{dy}{dt} = 3t^2 - 3 = 3(t^2 - 1) \Rightarrow \begin{cases} t = -1 \\ t = 1 \end{cases}$$

$$\frac{dx}{dt} = 2t$$

$$y = t^3 - 3t \Rightarrow y = 0 \Rightarrow t(t^2 - 3) = 0 \Rightarrow \begin{cases} t = 0 \\ t = \pm\sqrt{3} \end{cases}$$

$$m = \frac{3t^2 - 3}{2t} \Rightarrow \frac{6}{2\sqrt{3}} = \sqrt{3}, -\sqrt{3}$$

$$t = \sqrt{3} \Rightarrow x = 3, y = (\sqrt{3}^3 - 3\sqrt{3})$$

$$x = r(\theta - \sin(\theta))$$

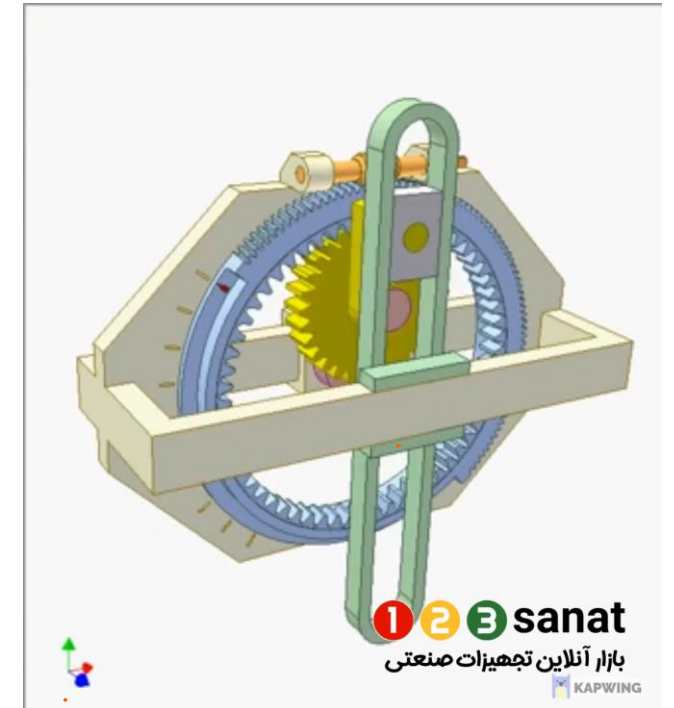
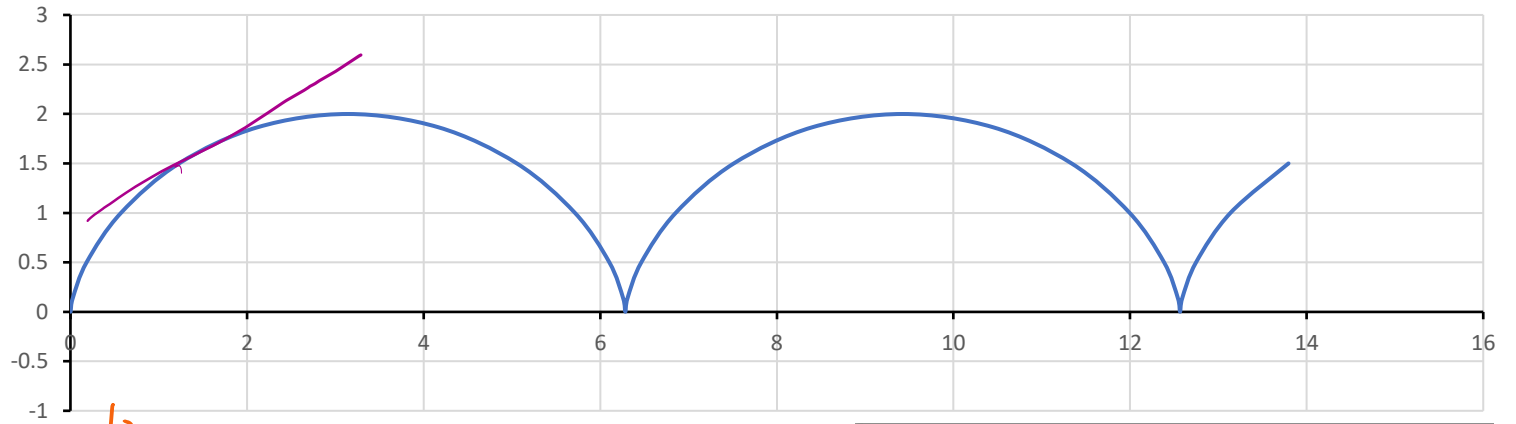
$$y = r(1 - \cos(\theta))$$

-sin θ

$$\theta = \frac{\pi}{3}$$

$$m = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r(\sin\theta)}{r(1 - \cos\theta)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}, \quad x, y$$

$$x = r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right), \quad y = r\left(\frac{1}{2}\right) \frac{r}{2} \quad y - \frac{r}{2} = \sqrt{3}\left(x - r\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right)$$



$$x = t^2 + 1$$

$$y = t^2 + t$$

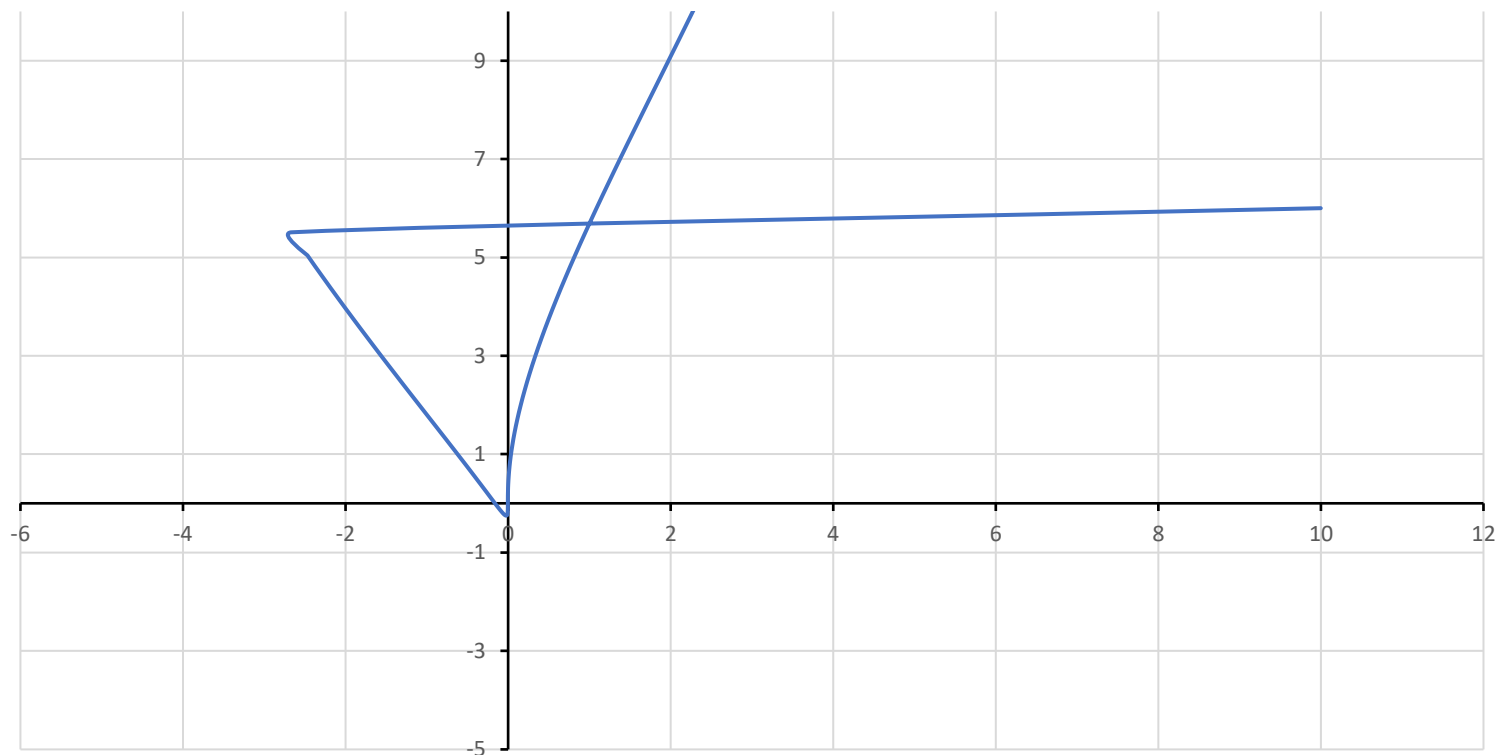
$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{2t}$$

$$\frac{d}{dt} \left(\frac{\frac{dy}{dx}}{\frac{dx}{dt}} \right)$$

$$\frac{4t - 4t}{2(2t)^2 - 2(2t+1)}$$

$$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{2}{8t^3} = \frac{1}{4t^3} \Rightarrow \text{مقل}$$



$$x = t^2$$

$$y = t^3 - 3t$$

$$\frac{d^2 y}{dx^2} = ? \quad \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\frac{dy}{dt} = 3t^2 - 3$$

$$\frac{dx}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$$

$$\frac{d}{dt} \rightarrow$$

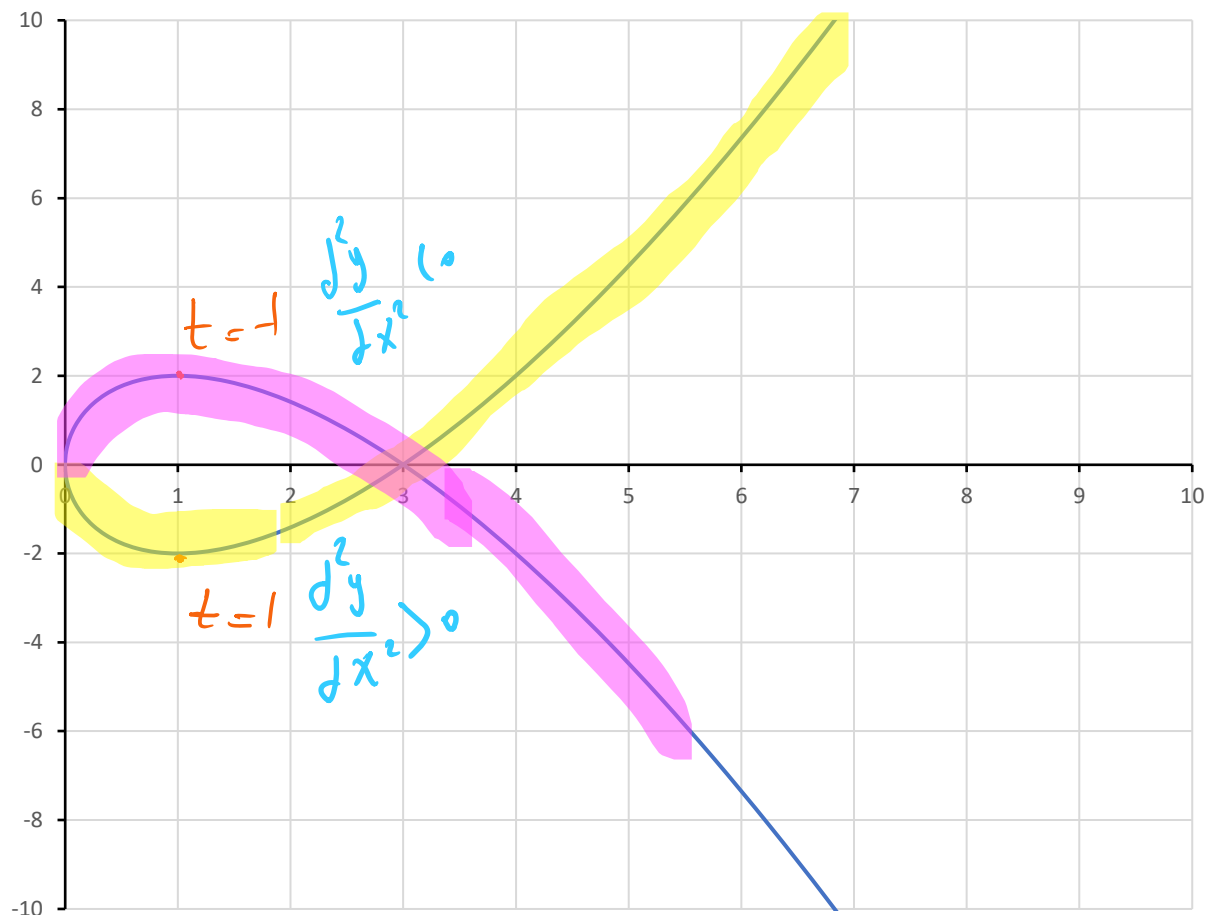
$$\frac{12t^2 - 6t^2 + 6}{4t^2}$$

$$= \frac{6t^2 + 6}{4t^2}$$

$$\frac{dx}{dt} = 2t$$

$$\frac{6t^2 + 6}{4t^2}$$

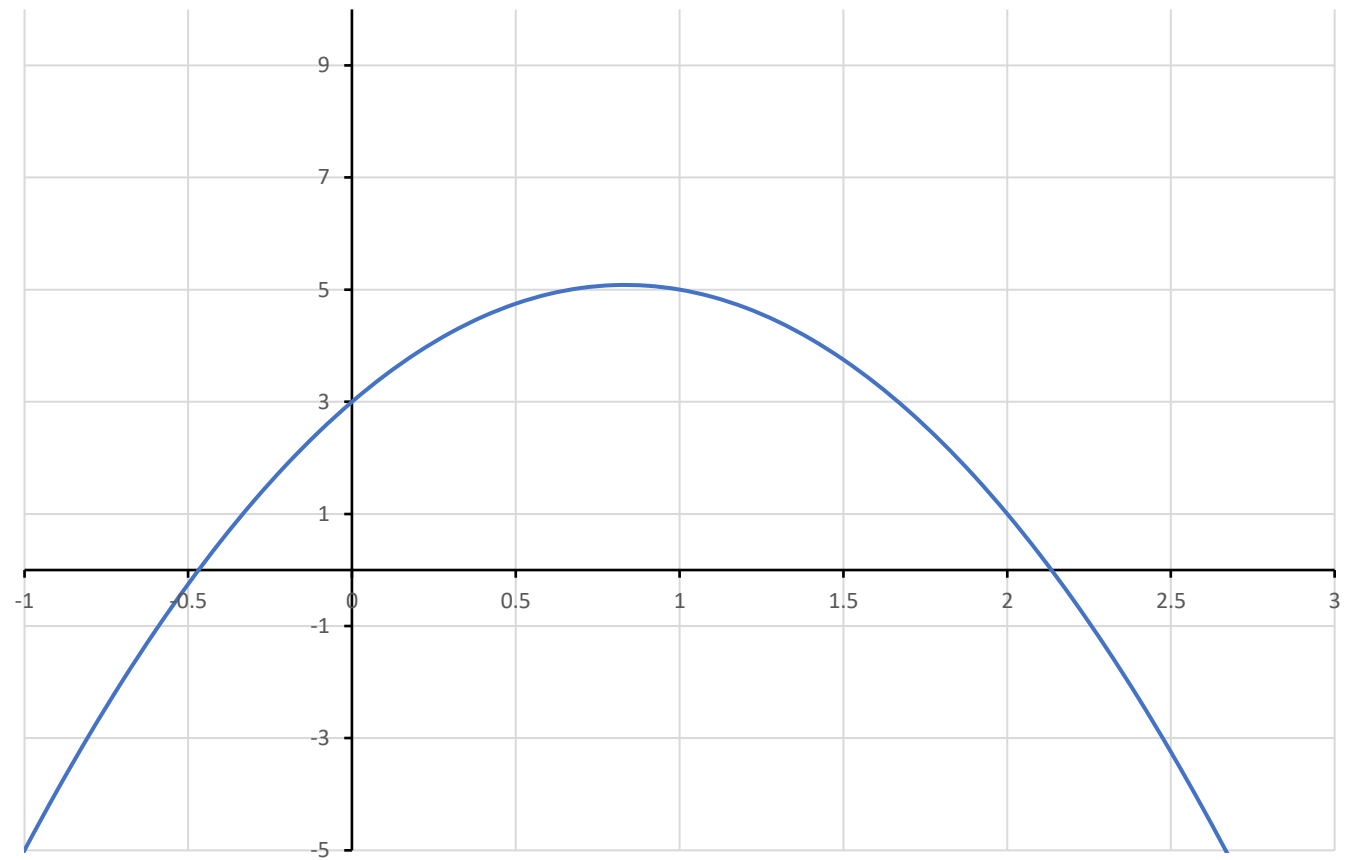
$$\frac{d^2 y}{dx^2} = \frac{\frac{6t^2 + 6}{4t^2}}{2t} = \frac{6(t^2 + 1)}{8t^3} = \frac{3(t^2 + 1)}{4t^3}$$



$$A = \int_a^b y \, dx = \int_\alpha^\beta g(t)f'(t)dt$$

$$x = f(t) \Rightarrow dx = f'(t)dt$$

$$y = g(t)$$



$$A = \int_a^b y \, dx = \int_\alpha^\beta \overbrace{g(t)}^{f(t)} f'(t) \, dt$$

$$x = t^3 + 1 \Rightarrow dx = 3t^2 \, dt \quad 0 < t < 2$$

$$y = 2t - t^2$$

$$A = \int_0^2 (2t - t^2) 3t^2 \, dt = 3 \int_0^2 (2t^3 - t^4) \, dt$$

$$= 3 \left[\frac{2t^4}{4} - \frac{t^5}{5} \right]_0^2 = \frac{24}{5}$$



$$x = r(\theta - \sin(\theta))$$

$$y = r(1 - \cos(\theta))$$

$$dx = r(1 - \cos \theta) d\theta$$

$$0 < \theta < 2\pi$$

$$A = \int y dx = \int_0^{2\pi} g(\theta) f'(\theta) d\theta$$

$$A = \int r^2 (1 - \cos \theta)(1 - \cos \theta) d\theta$$

$$A = r^2 \int_0^{2\pi} (1 - \cos \theta)^2 d\theta = r^2 \int_0^{2\pi} (1 + \cos^2 \theta - 2\cos \theta) d\theta = r^2 \left[\frac{3}{2} \theta + \frac{1}{2} \left(\frac{1}{2} \sin 2\theta \right) - 2\sin \theta \right]_0^{2\pi}$$

$$r^2 \int_0^{2\pi} \left[\frac{3}{2} + \frac{1}{2} \cos 2\theta - 2\cos \theta \right] d\theta$$

