به نام خدا

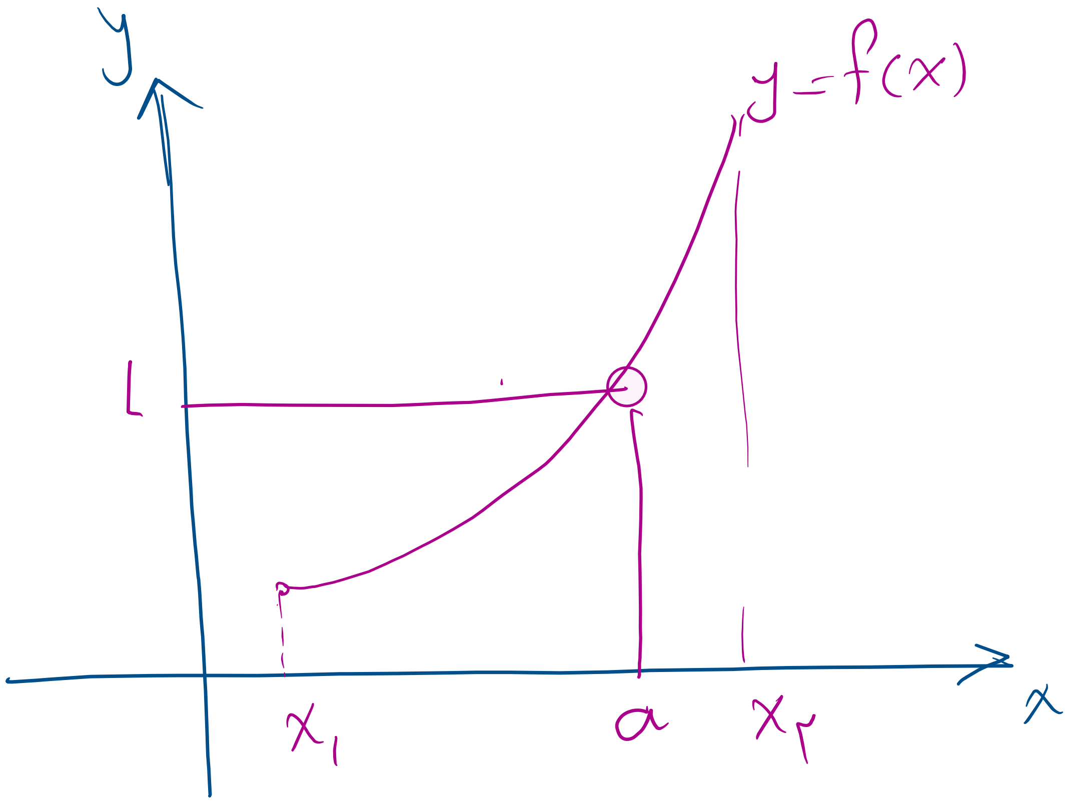
۱- حد
۲- پیوستگی
۳- مشتق
۴- انتگرال
۵- مختصات قطبی
۶- اعداد مختلط
۷- دنباله و سری

تاریخ امتحان = ۱۴۰۳/۶/۲۰

حضور و غیاب داخل اپلیکیشن هست

file → upload

۱- حد



$[x_۱ \;\; x_۲]$

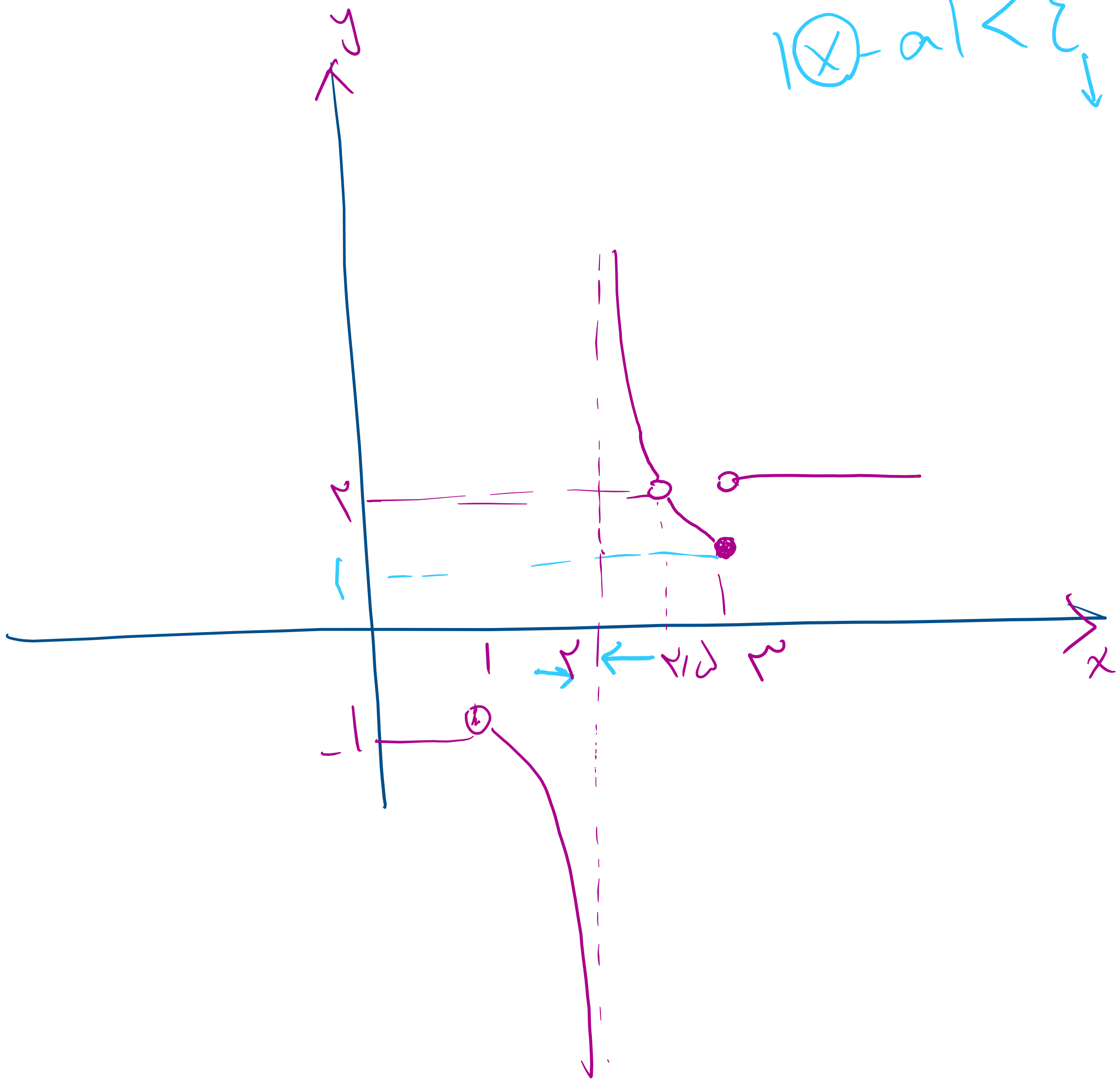
$$\lim_{x \to a} f(x) = l$$

$$\left.\begin{aligned} \lim_{x \to a^+} f(x) &= l_1 \\ \lim_{x \to a^-} f(x) &= l_2 \end{aligned}\right\} \; l_1 = l_2 \Rightarrow$$

حد تابع $f(x)$ در a برابر l هست

نقطه حدی

$|x-a|<\varepsilon$



$$\lim_{x\to 1} f(x) = ?$$

$$\lim_{x\to 2} f(x) = ? \begin{cases} x\to 2^+ = +\infty \\ x\to 2^- = -\infty \end{cases}$$ حد ندارد

$$\lim_{x\to 3} f(x) = \begin{cases} x\to 3^- = 1 \\ x\to 3^+ = 2 \end{cases}$$ حد ندارد

$$\lim_{x\to 2/5} f(x) = \begin{cases} x\to 2/5^- \Rightarrow f(x)=2 \\ x\to 2/5^+ \ \underline{f(x)=2} \end{cases}$$

حد برابر با ۲ است

$$\lim_{x \to 1} \sqrt{x-1} = ? \qquad \lim_{x \to 1^+} \sqrt{x-1} = 0$$

$[1 \quad \infty)$

قضایای حد

$$\lim_{x \to a} f(x) = \ell \Rightarrow \lim_{x \to a} c f(x) = c\ell$$

$$\lim_{x \to a} f(x)^n = \ell^n$$

$$\lim_{x \to a} f(x) = \ell_1$$

$$\lim_{x \to a} g(x) = \ell_2$$

$$\begin{cases} \lim_{x \to a} (f(x)+g(x)) = \ell_1 + \ell_2 \\ \lim_{x \to a} (f(x)-g(x)) = \ell_1 - \ell_2 \\ \lim_{x \to a} (f \cdot g)(x) = \ell_1 \times \ell_2 \\ \lim \left(\frac{f}{g}\right)(x) = \frac{\ell_1}{\ell_2} \end{cases}$$

$\ell_2 \neq 0$

$$\lim_{x \to 2} \frac{|x-2|}{x-2} = ?$$

$x \to 2^{+} \Rightarrow \lim_{x \to 2^{+}} \frac{\cancel{x-2}}{\cancel{x-2}} = 1$

$x \to 2^{-} \quad \lim_{x \to 2^{-}} \frac{-(\cancel{x-2})}{\cancel{x-2}} = -1$

تابع حد ندارد

$\lim_{x \to a} f(x) = l$

$\lim_{x \to a} g(x) \Rightarrow$ حد ندارد

$\Rightarrow$

$\lim (f+g)(x) = ?$ حد ندارد

$\lim (f-g)(x) = ?$ 〃

$\lim \left(\frac{g}{f}\right)(x) = ?$ 〃

$\lim \left(\frac{f}{g}\right)(x) = ?$

$\lim (f \cdot g)(x) = ?$

ممکن است داشته باشد

نداشته باشد — —

قضیه فشردگی یا ساندویچ

$$f(x) \leq g(x) \leq h(x)$$

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = l \Rightarrow \lim_{x \to a} g(x) = l$$

مثال

$$\lim_{x \to 0} x \sin\frac{1}{x} = ?$$

$$-1 \leq \sin\frac{1}{x} \leq 1 \Rightarrow -x \leq x \sin\frac{1}{x} \leq x$$

$$\underbrace{\lim_{x \to 0} \overbrace{-x}^{f(x)}}_{0} \leq \underline{\lim_{x \to 0} \overbrace{x \sin\frac{1}{x}}^{g(x)}} \leq \underbrace{\lim_{x \to 0} \overbrace{x}^{h(x)}}_{0}$$

حد برابر با صفر است

قضیه کران‌دار

$$\left.\begin{array}{l}\lim\limits_{x\to a} f(x)=0\\[2mm] \lim\limits_{x\to a} g(x)=\text{کران‌دار}\end{array}\right\}\ \lim_{x\to a} f(x).g(x)=0$$

مثال

$$\lim_{x\to 0} \underbrace{x}_{\text{صفر}}\,\sin\frac{1}{x}=?$$

x: $f(x)$ ، $\sin\frac{1}{x}$: $g(x)$

تابع $g(x)$ در همسایگی a کران‌دار است

$$\Rightarrow \lim f(x).g(x)=0$$

تمرین

$$\lim_{x\to 0} \tan x \sin\frac{1}{x}=?$$

$$\lim_{x\to 0} x\sqrt{\sin^2 x}=?$$

$$\lim_{x\to 0^+} \frac{x+x^2}{|x|} = \lim \frac{\cancel{x}(1+x)}{\cancel{x}} = \lim_{x\to 0^+} 1+x = 1 \Rightarrow \lim_{x\to 0} \frac{x+x^2}{|x|} = ?$$

تمرین

$$\lim_{x\to 0} 5^{-\frac{1}{x^2}} = ? \quad -\infty \quad 0$$

$$\lim_{x\to -2^-} \frac{x^2-4}{x+2} \underbrace{\operatorname{sgn}(x+2)}_{-1} = \lim_{x\to -2^-} \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}(-1) = \lim_{x\to -2^-} 2-x = 4$$

$$\lim_{x\to 0} \frac{|3x-1|-|3x+1|}{x} = \lim \frac{-(3x-1)-(3x+1)}{x} = \lim_{x\to 0} \frac{-6\cancel{x}}{\cancel{x}} = -6$$

مثال)

$$f(x)=\begin{cases}\dfrac{2x+4}{x+1} & x\in\mathbb{Z} \quad \times \\[2ex] \dfrac{5}{x^2+1} \ \checkmark & x\notin\mathbb{Z} \quad \checkmark\end{cases}$$

$$\lim_{x\to 1} f(x)=\lim_{x\to 1}\frac{5}{x^2+1}=2,5$$

$$f(x)=\begin{cases}\sqrt{1-x} & x>0 \ \checkmark \\ -\sqrt{1+x} & x<0\end{cases}$$

$$\lim_{x\to 0^-} f(x^3-x)=\ ?$$

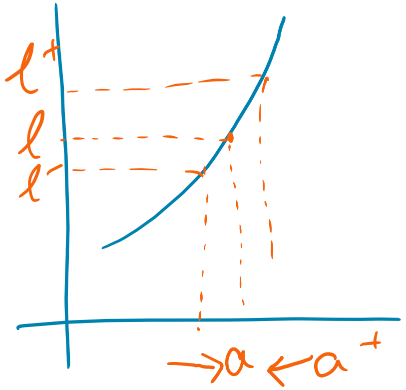
$$x^3-x \Rightarrow \underset{-}{x}\,\underbrace{(x^2-1)}_{-}=+$$

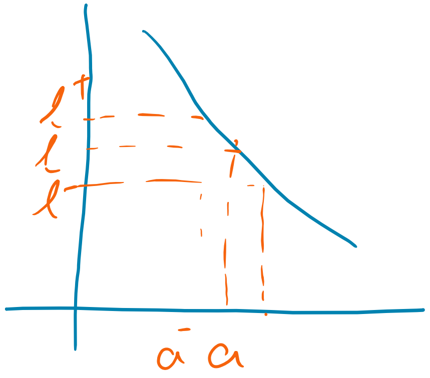
$$\lim_{x\to 0^-} f(x^3-x)=+1$$

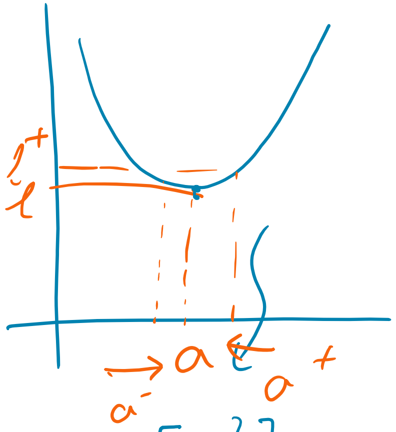
$$\lim_{x\to 3}[x]=\begin{cases}x\to 3^+ \Rightarrow \lim f(x)=3 \\ x\to 3^- \Rightarrow \lim f(x)=2\end{cases}$$ ندارد

$$\lim_{x\to 3/2} f(x)=$$

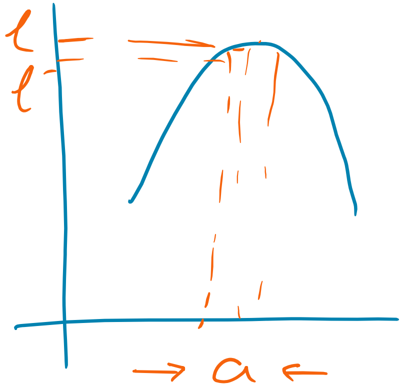
$$\lim_{x \to a} [f(x)] = l \qquad a \begin{cases} \in \mathbb{Z} \Rightarrow \lim f(x) \Rightarrow \text{ندارد} \\ \notin \mathbb{Z} \Rightarrow \lim f(x) = \text{حد دارد} \end{cases}$$







$[x^2]$



به طور کلی زمانی که داخل براکت مقدار صحیح شود تابع حد ندارد مگر اینکه در آن نقطه تابع اکسترمم باشد

$$f(x) = [x^2] \qquad \lim_{x \to 0} f(x) = \begin{cases} 0^+ \to 0 \\ 0^- \to 0 \end{cases}$$

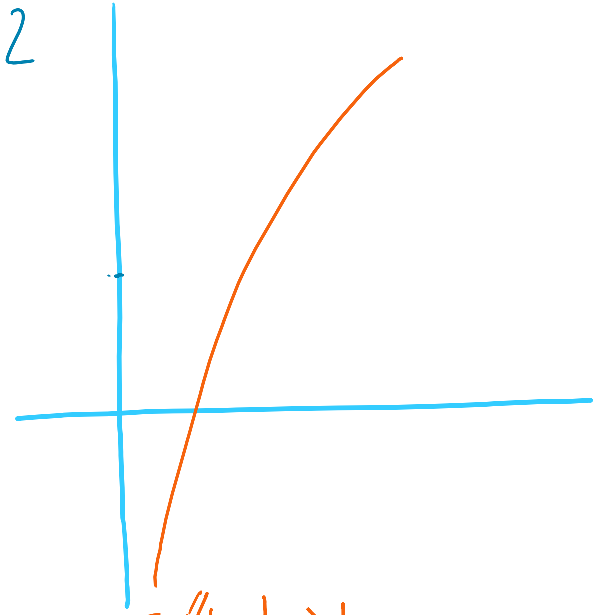
$$\underbrace{(-2, 2)}$$

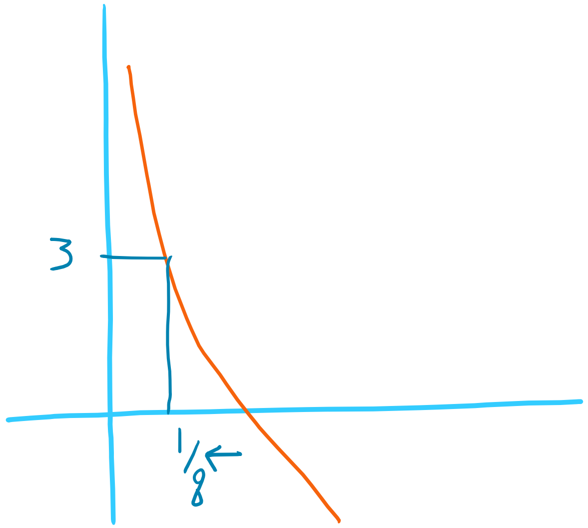
در چند نقطه حد ندارد

$$x^2 = k \Rightarrow x = \pm\sqrt{k} \qquad k \in \mathbb{Z}$$

$$\{-\sqrt{3}, -\sqrt{2}, -\sqrt{1}, \cancel{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}\}$$

$$\lim_{x\to \frac{1}{8}^+} \left[\log_{\frac{1}{2}} x\right] = ? \qquad [3^-]=2$$





هرگاه در محاسبه حد یک تابع کسری، مخرج کسر صفر مطلق باشد، جواب حد الزاما وجود ندارد است

4

4^-

$\frac{1}{2}$

$$\left[\lim_{x\to \left(\frac{\pi}{6}\right)^-} 8^{\sin x}\right] = ? \quad 4$$

* تمرین $\lim_{x\to \frac{1}{5}} [5x]+[-5x] =$

الزاما حد ندارد

$$\lim_{x\to 3^+} \frac{x+1}{\underbrace{[x-3]}_{\varepsilon^+}} = ? \ \checkmark$$

$$\lim_{x\to 0} \frac{1+\sin x}{[x^2]}$$

$$\lim \frac{1-[x]^2}{1-[x]} = ? \begin{cases} x \to 1^+ & \text{حد ندارد} \\ x \to 1^- & 1 \end{cases}$$

$$\lim_{x \to 3^-} \frac{|x-3|}{x-3}[x] = ? \qquad \frac{-(\cancel{x-3})}{\cancel{x-3}} \underbrace{[3^-]}_{2} = -2$$

$$\lim_{x \to -1^+} \frac{|x|}{[x]} \operatorname{sgn}(x+1) = ? \qquad -1$$

$$f(x) = \left[\frac{4x^2+\overset{4}{\overline{3+1}}-1}{x^2+1}\right] = \left[\frac{4(x^2+1)-1}{x^2+1}\right] = \left[4 - \frac{1}{x^2+1}\right] = 4 + \underbrace{\left[\frac{-1}{x^2+1}\right]}_{-1} = 3$$

تابع در تمامی

نقاط حد دارد

$$x^2 \geq 0 \qquad x^2+1 \geq 1 \Rightarrow 0 < \frac{1}{x^2+1} \leq 1 \Rightarrow -1 \leq \frac{-1}{x^2+1} < 0$$

$$\lim_{x \to 1^+} \frac{\overset{0^+}{\overbrace{[x-1]}}}{x-[x]} = ?$$

$$\frac{\text{مطلق } 0}{\text{حدی } 0} = 0$$

$$\lim_{x \to 0} \frac{[x^2]-x^2}{x^2} = ?$$

$$\lim_{x \to 0^+} \frac{1-2\cos x}{3^x-1} = ?$$

$$\lim_{x \to \frac{\pi}{4}^+} \sqrt[4]{\cot x - 1}$$

$$f(x) = \frac{(3-[x])\sqrt{x^2-4x+4}}{x^2-4} \qquad \lim_{x \to 2} f(x)$$

صورت‌های مبهم

$$\frac{0}{0} \qquad \frac{\infty}{\infty} \qquad \infty-\infty \qquad 0\times\infty \qquad 1^{\infty} \qquad 0^{0} \qquad \infty^{0}$$

1- $\frac{0}{0}$

برای رفع ابهام:

1- هوپیتال ← مشتق

2- حذف عامل صفرکننده ✓

3- هم‌ارزی ✓

$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}=\frac{0}{0} \qquad \sin^2 x = 1-\cos^2 x$$

$$\lim_{x\to\pi}\frac{\cancel{(1+\cos x)}(1+\cos^2 x-\cos x)}{\cancel{(1+\cos x)}(1-\cos x)}=\frac{3}{2}$$

$$\lim_{x\to\pi^+}\frac{\sqrt{1+\cos x}}{\sin x}=\frac{0}{0} \qquad -\frac{\sqrt{2}}{2}$$

$$1+\cos x = 2\cos^2\frac{x}{2}$$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\lim_{x\to\pi}\frac{\sqrt{2\cos^2\frac{x}{2}}}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\frac{\left|\cos\frac{x}{2}\right|\times\sqrt{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\frac{-\sqrt{2}\,\cancel{\cos\frac{x}{2}}}{2\sin\frac{x}{2}\,\cancel{\cos\frac{x}{2}}}$$

$$\Rightarrow \lim_{x\to\pi}\frac{-\sqrt{2}}{2\sin\frac{x}{2}}=-\frac{\sqrt{2}}{2}$$

$$\lim_{x \to 1^+} \frac{(x-1)\sqrt{x-1} + x^2 - 1}{x^3 - 1} = \lim \frac{(x-1)\sqrt{x-1} + (x-1)(x+1)}{(x-1)(x^2+1+x)} = \lim_{x \to 1^+} \frac{\cancel{(x-1)}\left[\sqrt{x-1} + x + 1\right]}{\cancel{(x-1)}(x^2+1+x)} = \frac{2}{3}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{\sin^2 x\,(1-\sin x)} = \lim_{x \to \frac{\pi}{2}} \frac{\cancel{(1-\sin x)}(1+\sin x)}{\sin^2 x\,\cancel{(1-\sin x)}} = \frac{2}{1} = 2$$

$1 - \sin^2 x$

هم‌ارزی

$$\underbrace{f(x) \sim g(x)}_{x=a \text{ نزدیک}} \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = 1$$

$$x \to 0 \begin{cases} \sin x \sim x \\ \tan x \sim x \\ \cos x \sim 1 - \frac{x^2}{2} \Rightarrow \boxed{1 - \cos x \sim \frac{x^2}{2}} \end{cases}$$

2

2

$$\lim_{x \to 0} \frac{\sin x}{x} = ? \quad \frac{0}{0} \Rightarrow \lim_{x \to 0} \frac{x}{x} = 1$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin(\overbrace{\cot x}^{u})}{\cot x} = \frac{0}{0} \Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{\cot x}{\cot x} = 1$$

$$\lim_{x \to 0} \frac{\tan x \cdot \sin 2x \cdot \sin 3x}{3x^2 \sin x} = \frac{x \times 2x \times 3x}{3x^2 \times x} = 2$$

$$\lim_{x \to 0} \frac{\sin^2 x}{\underbrace{1 - \cos 2x}} = \lim_{x \to 0} \frac{x^2}{\frac{(2x)^2}{2}} = \frac{1}{2}$$

$$\lim \frac{\cos 2x - 1}{\underbrace{\sin^2 x}} = -2$$

$$1 - \cos x \sim \frac{x^2}{2}$$

$$\lim_{x \to 0} \frac{1-\cos 2x}{\tan^2 \frac{x}{2}} = 8$$

$$1-\cos u \sim \frac{u^2}{2} \quad \Rightarrow \quad \lim_{x \to 0} \frac{\frac{(2x)^2}{2}}{\left(\frac{x}{2}\right)^2} = 8$$

$$\tan u \sim u$$

$$\lim_{x \to \frac{\pi}{4}} \frac{1-\cos\left(x-\frac{\pi}{4}\right)}{\tan^2\left(x-\frac{\pi}{4}\right)} = \lim_{x \to \frac{\pi}{4}} \frac{\frac{\left(x-\frac{\pi}{4}\right)^2}{2}}{\left(x-\frac{\pi}{4}\right)^2} = \frac{1}{2}$$

u

$$\lim_{x \to 0} \frac{\sin(1-\cos x)}{x^2} = ? \qquad 1/2$$

$$\lim_{x \to 1} \frac{1-x^2}{(\arccos x)^2} = ? = \lim_{\alpha \to 0} \frac{1-\cos^2\alpha}{\alpha^2} = \frac{\sin^2\alpha}{\alpha^2} = \frac{\alpha^2}{\alpha^2} = 1$$

$\alpha \to 0$

$$\arccos x = \alpha$$

$$\cos\alpha = x$$

$$\lim_{x \to 0} \frac{\cos x - \cos 3x}{x^2} = 4$$

$$\boxed{\cos^K u \sim 1 - \frac{K u^2}{2}}$$

$$\lim_{x \to 0} \frac{1-\cos^3 x}{(1-\tan x)x} = \lim \frac{1-\left(1-\frac{3x^2}{2}\right)}{(1-x)\cancel{x}} = \frac{\frac{3x}{2}}{1-x} = \lim_{x \to 0} \frac{3x}{2(1-x)} = 0$$

$$\lim_{x \to 0} \frac{x^3 - \sin^3 x}{x^5} = ?$$

بسط مکلورن

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\tan x = x + \frac{x^3}{3}$$

$$\cos x = x - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

$$\lim_{x\to 0}\frac{x^3-\sin^3 x}{x^5}=? \quad \frac{(x-\overset{x-\frac{x^3}{6}}{\widehat{\sin x}})(x^2+\overset{\sim x^2}{\overline{\sin^2 x}}+\overset{x^2}{\overline{x\sin x}})}{x^5}=\frac{(\cancel{x}-\cancel{x}+\frac{x^3}{6})(3x^2)}{x^5}=\lim\frac{\frac{1}{2}x^5}{x^5}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2} \qquad \lim\frac{x+\frac{x^3}{3}-\left(x-\frac{x^3}{6}\right)}{x^3}=\frac{1}{2}$$

$$\lim_{x\to 0}\frac{\sin x^3-x^3}{\tan^9 x}=-\frac{1}{6} \qquad \lim_{x\to 0}\frac{\sin^2 x-x^2\cos^2 x}{x^2\sin^2 x}=\frac{2}{3}$$

$$\lim_{x\to 0}\frac{\sin x-x+\frac{x^3}{6}}{x^5}=?\ \frac{1}{120}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x}{7!} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots$$

$$\cos x = x - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

* تمرین

$$\lim \frac{\sinh x - \tan x}{x\left(e^{x^2}-1\right)} =$$

$$\lim \frac{e^x - \sin x + \cos x - 2}{\sqrt[3]{1+x^2} - 1} =$$

$$\lim_{x \to 0} \frac{e^x \overbrace{-1-\sin x}^{\text{دو ترم}}}{x \sin x} = \frac{1}{2}$$

$$e^x \approx \underbrace{1 + x + \frac{x^2}{2!}} + \frac{x^3}{3!} + \cdots$$

صورت ابهام $\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x}}}{\sqrt{5x+3}} = ? \qquad \frac{\infty}{\infty} \qquad \frac{\cancel{\sqrt{x}}}{\underbrace{\sqrt{5x}}_{\sqrt{5}\times\cancel{\sqrt{x}}}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\lim_{x\to-\infty} \frac{5x+\sqrt[3]{x}}{\sqrt{x^2+3x}+4x} = ?\ \lim_{x\to-\infty} \frac{5x}{\underbrace{|x|}_{-x}+4x} = \lim_{x\to-\infty} \frac{5x}{3x} = \frac{5}{3}$$

$$\lim_{x\to\infty} \frac{x^n}{e^x} = ? \qquad \frac{\infty}{\infty\uparrow} = 0$$

قوانین رشد

$$\boxed{\begin{array}{l} b>a>1 \quad \alpha,\beta>0 \quad x\to\infty \\ b^x \gg a^x \gg x^\alpha \gg (\ln x)^\beta \end{array}}$$

$x\to\infty$

$$\lim_{x\to\infty} \frac{e^x}{x^3} = \frac{\infty}{\infty} \Rightarrow \infty$$ قوانین رشد

$$\lim_{x\to\infty} \frac{\ln x}{x} = 0$$

قوانین رشد

$$\boxed{\begin{array}{l} b>a>1 \quad \alpha,\beta>0 \quad x\to\infty \\ b^x \gg a^x \gg x^\alpha \gg (\ln x)^\beta \end{array}}$$

$$\checkmark \lim \frac{x^6+4^{x+1}+2^{x-1}}{4^{x-1}+3^x+\log x} = \frac{4^{x+1}}{4^{x-1}} = 4^2 = 16$$

$$\lim_{x\to\frac{1}{2}} \frac{\cot\left(x-\frac{1}{2}\right)}{\tan \pi x} = \frac{\infty}{\infty}$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty} \Rightarrow \lim_{x\to a} \frac{\frac{1}{g(x)}}{\frac{1}{f(x)}} = \frac{0}{0}$$

$$\lim \frac{\frac{1}{\tan \pi x}}{\frac{1}{\cot\left(x-\frac{1}{2}\right)}} = \frac{0}{0} \Rightarrow \lim \frac{\cot \pi x}{\tan\left(x-\frac{1}{2}\right)} = \frac{0}{0} \Rightarrow$$ حل به عهده خودتان

* $$\lim \frac{xe^{2x}}{e^{3x}+x^{100}-\log x} = ?$$

* $$\lim_{x \to 0} \frac{1+2^{\frac{1}{x}}}{3+2^{\frac{1}{x}}}$$

* $$\lim_{x \to 0^+} \frac{\ln(\tan 2x)}{\ln(\tan 3x)}$$

رفع ابهام $\infty - \infty$

$$\lim_{x\to -2}\left(\frac{x+5}{(x-1)(x+2)}+\frac{1}{x+2}\right)^{\infty-\infty} = \lim_{x\to -2}\left(\frac{x+5+x-1}{(x-1)(x-2)}\right) = \lim \frac{\overset{2(\cancel{x+2})}{2x+4}}{(x-1)\cancel{(x+2)}} =$$

$$\lim_{x\to -2}\frac{2}{x-1} = -\frac{2}{3}$$

$$\lim_{x\to +\infty}\left(x-\sqrt{x^2+1}\right) = \infty - \infty \Rightarrow \lim\left(x-\sqrt{x^2+1}\right)\times\frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2+1}} = \frac{x^2-(x^2+1)}{x+\sqrt{x^2+1}} = \lim_{x\to\infty}\frac{-1}{x+\sqrt{x^2+1}} = 0$$

$$\lim_{x\to+\infty}\left(\sqrt{\underbrace{4}_{a}x^2+\underbrace{4}_{b}x+1}-\sqrt{\underbrace{4}_{a}x^2-\underbrace{16}_{b}x+2}\right)$$

$$\lim_{x\to\pm\infty}\sqrt[n]{ax^n+bx^{n-1}+\cdots}\sim\begin{cases}\sqrt[n]{a}\left|x+\dfrac{b}{na}\right| & \text{زوج } n\\[2ex] \sqrt[n]{a}\left(x+\dfrac{b}{na}\right) & \text{فرد } n\end{cases}$$

$$\sqrt{4}\left|x+\frac{4}{2\times4}\right|-\sqrt{4}\left|x-\frac{16}{2\times4}\right|=2\left|x+\frac{1}{2}\right|-2\left|x-2\right|=2\left(x+\frac{1}{2}\right)-2(x-2)=5$$

رفع ابهام $0\times\infty$

$$\lim_{x\to a}f(x)\cdot\underbrace{g(x)}_{\infty\text{ حالت}}=0\times\infty\Rightarrow\lim_{x\to a}\frac{f(x)}{\dfrac{1}{g(x)}}=\frac{0}{0}$$

$$\lim_{x\to\infty}\underbrace{\sin\frac{1}{3\sqrt{x}}}_{f(x)}\underbrace{\cot\left(\frac{3\sqrt{x}}{x+5}\right)}_{\infty}\qquad 0\times\infty$$

$$\frac{1}{\cot\left(\dfrac{3\sqrt{x}}{x+5}\right)}\longrightarrow\tan\frac{3\sqrt{x}}{x+5}$$

$$\lim\frac{\sin\dfrac{1}{3\sqrt{x}}}{\tan\dfrac{3\sqrt{x}}{x+5}}=\frac{\dfrac{1}{3\sqrt{x}}}{\dfrac{3\sqrt{x}}{x+5}}=\lim_{x\to\infty}\frac{x+5}{9x}=\frac{\infty}{\infty}=\frac{x}{9x}=\frac{1}{9}$$

رفع ابهام 1^{∞}

$$\lim_{x \to 0} (\sin 4x \cot 2x - \sin 4x \cot x) = \frac{-2}{} \checkmark$$

$$\lim_{x \to a} f(x)^{g(x) \to \infty} = 1 \Rightarrow \lim_{x \to a} e^{(f(x)-1)g(x)}$$

$\pm\infty$ میتوانند

$$\lim_{x \to 0} \underbrace{(1+x)}_{f(x)}^{g(x)=\frac{1}{x} \to \infty} = 1 \Rightarrow \lim_{x \to 0} e^{(1+x-1)\frac{1}{x}} = e$$

$$\lim_{x \to \infty} \left(1+\frac{a}{x}\right)^{x} = 1^{\infty} \Rightarrow e^{a}$$

$$\lim_{x \to 0} (1-4x)^{\left(\frac{1-x}{x}\right)} \Rightarrow e^{-4}$$

$$\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x} = 1^{\infty} \Rightarrow \frac{1}{e}$$

$$\lim_{x \to \infty} \left(\frac{x^2+1}{x^2-1}\right)^{x^2} = 1^{\infty} \Rightarrow e^2$$

$$\lim_{x \to 0} \left(\cos 5x\right)^{\frac{2}{x^2}} \Rightarrow e^{-5}$$

$$f(x) = g(x) = [1 - x^2]$$

$$\lim_{x \to 1} (g \circ f)(x) = ?$$

$$\lim_{x \to 1} [\overbrace{1 - x^2}^{f(x)}] \begin{cases} x \to 1^+ \Rightarrow \lim [0^-] = -1 \Rightarrow g \circ f(x) = 0 \\ x \to 1^- \Rightarrow \lim [0^+] = 0 \Rightarrow g \circ f(x) = 1 \end{cases}$$