

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

۱- حد

۲- پیوستگی

۳- مشتق

۴- انتگرال

۵- نقاط تقاطع

۶- اعداد مختلط

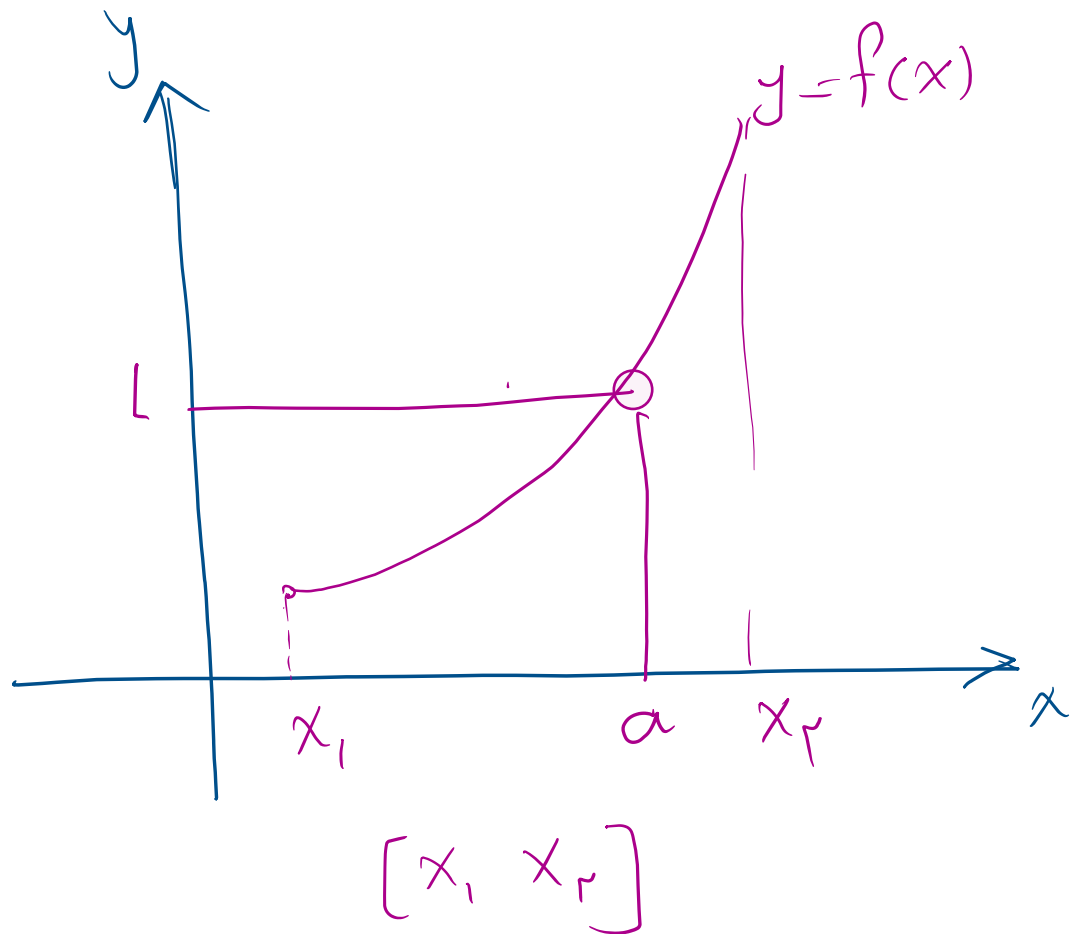
۷- دنباله و سری

۶/۲۰/۱۴۰۳ = تاریخ امتحان

حضور و غیاب داخل المپیک هست

file → upload

الحل

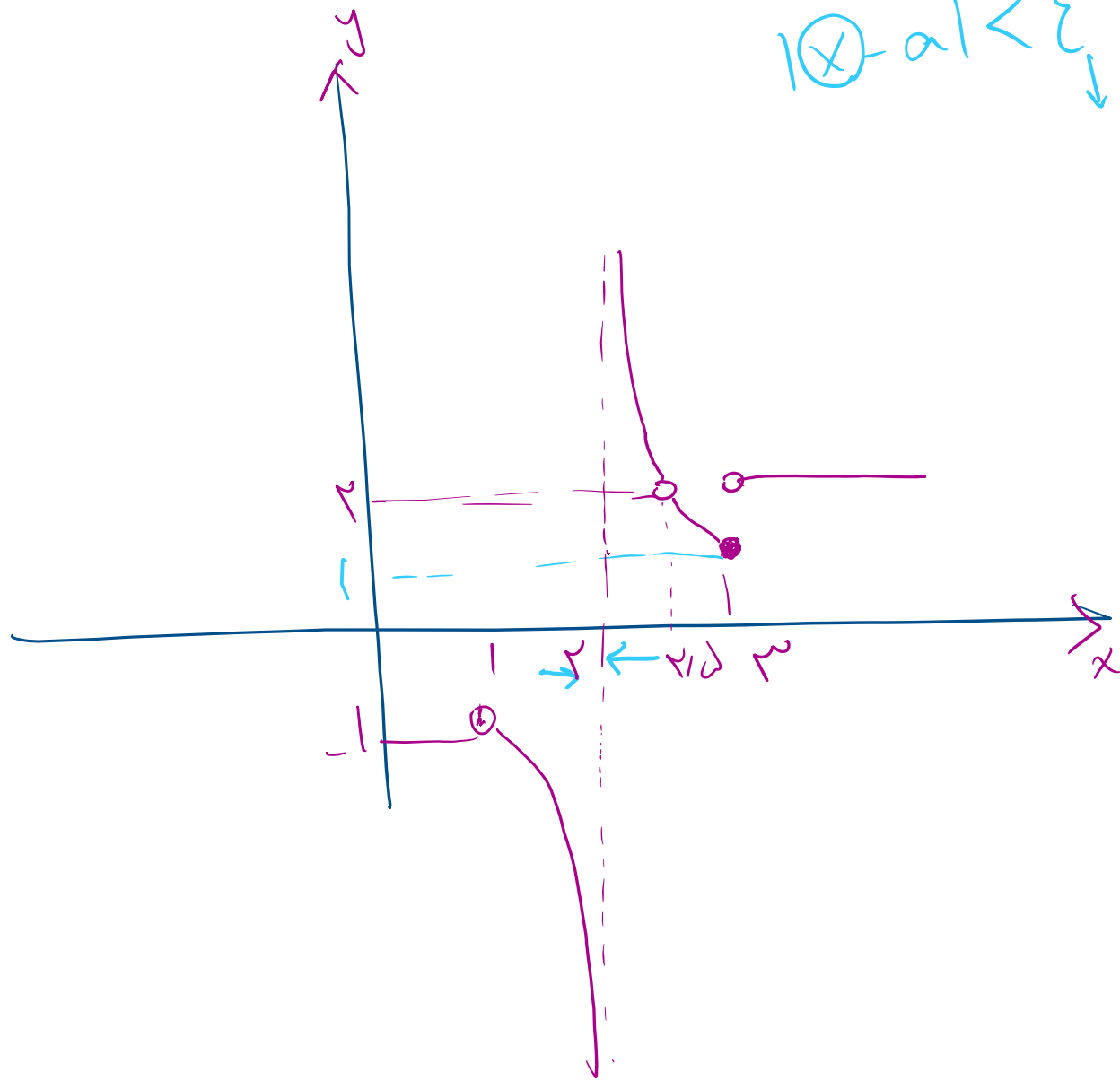


$$\lim_{x \rightarrow a} f(x) = l$$

$$\lim_{x \rightarrow a^+} f(x) = l_1$$

$$\lim_{x \rightarrow a^-} f(x) = l_2$$

$\left. \begin{array}{l} \lim_{x \rightarrow a^+} f(x) = l_1 \\ \lim_{x \rightarrow a^-} f(x) = l_2 \end{array} \right\} l_1 = l_2 \Rightarrow f(x) \text{ تتقارب نحو } a, \text{ فتساوي } l$



$$1 \otimes a < \varepsilon$$

تعریف

$$\lim_{x \rightarrow 1} f(x) = ?$$

$$\lim_{x \rightarrow 2} f(x) = ? \begin{cases} x \rightarrow 2^+ = +\infty \text{ / } \text{حد} \\ x \rightarrow 2^- = -\infty \end{cases}$$

$$\lim_{x \rightarrow 3} f(x) = \begin{cases} x \rightarrow 3^- = 1 \\ x \rightarrow 3^+ = 2 \end{cases} \text{ / } \text{حد}$$

$$\lim_{x \rightarrow 2} f(x) = \begin{cases} x \rightarrow 2^- \Rightarrow f(x) = 2 \\ x \rightarrow 2^+ \Rightarrow f(x) = 2 \end{cases}$$

حد برابر 2 است

$$\lim_{\substack{x \rightarrow 1 \\ [1, \infty)}} \sqrt{x-1} = ? \quad \lim_{x \rightarrow 1^+} \sqrt{x-1} = 0$$

قضايا

$$\lim_{x \rightarrow a} f(x) = l \Rightarrow \begin{aligned} \lim_{x \rightarrow a} C f(x) &= C l \\ \lim_{x \rightarrow a} f(x)^n &= l^n \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow a} f(x) &= l_1 \\ \lim_{x \rightarrow a} g(x) &= l_2 \end{aligned} \left\{ \begin{aligned} \lim_{x \rightarrow a} (f(x) + g(x)) &= l_1 + l_2 \\ \lim_{x \rightarrow a} (f(x) - g(x)) &= l_1 - l_2 \\ \lim_{x \rightarrow a} (f \cdot g)(x) &= l_1 \times l_2 \\ \lim_{x \rightarrow a} \left(\frac{f}{g} \right)(x) &= \frac{l_1}{l_2} \end{aligned} \right.$$

$l_2 \neq 0$

$$\lim_{x \rightarrow 2} \frac{|x-2|}{x-2} = ?$$

$$x \rightarrow 2^+ \Rightarrow$$

$$\lim_{x \rightarrow 2^+} \frac{x-2}{x-2} = 1$$

$$x \rightarrow 2^+$$

$$\lim_{x \rightarrow 2^-} \frac{-(x-2)}{x-2} = -1$$

$$x \rightarrow 2^-$$

تابع حد ندارد

$$\lim_{x \rightarrow a} f(x) = l$$

$$\lim_{x \rightarrow a} g(x) \Rightarrow \text{حد دارد}$$

$\Rightarrow$

$$\lim (f+g)(x) = ? \quad \text{حد دارد}$$

$$\lim (f-g)(x) = ? \quad //$$

$$\lim \left(\frac{g}{f} \right)(x) = ? \quad //$$

$$\lim \left(\frac{f}{g} \right)(x) = ?$$

$$\lim (f \cdot g)(x) = ?$$

ممكن ان تكون f و g متساويين
ممكن ان تكون f و g مختلفين

قضیه فشردگی با ساندیج

$$\overbrace{f(x)} \leq \overbrace{g(x)} \leq \overbrace{h(x)}$$

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = l \Rightarrow \lim_{x \rightarrow a} g(x) = l$$

مثال

$$\lim_{x \rightarrow 0} \overbrace{x \sin \frac{1}{x}} = ?$$

$$\begin{array}{c} -1 < \underbrace{\sin \frac{1}{x}}_{f(x)} \leq 1 \Rightarrow -x \leq \underbrace{x \sin \frac{1}{x}}_{g(x)} \leq \underbrace{x}_{h(x)} \\ \lim_{x \rightarrow 0} \underbrace{-x}_{f(x)} \leq \lim_{x \rightarrow 0} \underbrace{x \sin \frac{1}{x}}_{g(x)} \leq \lim_{x \rightarrow 0} \underbrace{x}_{h(x)} \\ \underbrace{\quad\quad\quad}_0 \qquad\qquad\qquad \underbrace{\quad\quad\quad}_0 \qquad\qquad\qquad \underbrace{\quad\quad\quad}_0 \end{array}$$

حد برابر با صفر است

قضیه کیدان داره

$$\lim_{x \rightarrow a} f(x) = 0$$

$$\lim_{x \rightarrow a} g(x) = \text{محدود}$$

$$\lim_{x \rightarrow a} f(x) \cdot g(x) = 0$$

مثال

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x} = ?$$

تابع $g(x) = \sin \frac{1}{x}$ در $a=0$ محدود است

$$\Rightarrow \lim_{x \rightarrow 0} f(x) \cdot g(x) = 0$$

تمرین

$$\lim_{x \rightarrow 0} \tan x \sin \frac{1}{x} = ?$$

$$\lim_{x \rightarrow 0} x \sqrt{\sin^2 x} = ?$$

$$\lim_{x \rightarrow 0^+} \frac{x+x^2}{|x|} = \lim_{x \rightarrow 0^+} \frac{x(1+x)}{x} = \lim_{x \rightarrow 0^+} 1+x = 1 \Rightarrow \lim_{x \rightarrow 0} \frac{x+x^2}{|x|} = ? \quad \text{تمرین}$$

$$\lim_{x \rightarrow 0} \omega \left(\frac{-1}{x^2} \right) = ? \quad \infty$$

$$\lim_{x \rightarrow -2^-} \frac{x^2-4}{x+2} \underbrace{\text{sgn}(x+2)}_{-1} = \lim_{x \rightarrow -2^-} \frac{(x-2)(\cancel{x+2})}{\cancel{x+2}} (-1) = \lim_{x \rightarrow -2^-} 2-x = 4$$

$$\lim_{x \rightarrow 0} \frac{|3x-1| - |3x+1|}{x} = \lim_{x \rightarrow 0} \frac{-(3x-1) - (3x+1)}{x} = \lim_{x \rightarrow 0} \frac{-6x}{x} = -6$$

lim)

$$f(x) = \begin{cases} \frac{2x+4}{x+1} & x \in \mathbb{Z} \times \\ \frac{5}{x^2+1} & \checkmark x \notin \mathbb{Z} \checkmark \end{cases}$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{5}{x^2+1} = 2,5$$

$$f(x) = \begin{cases} \sqrt{1-x} & x > 0 \checkmark \\ -\sqrt{1+x} & x < 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x^3 - x) = ?$$

$$x^3 - x \Rightarrow x(x^2 - 1) = +$$

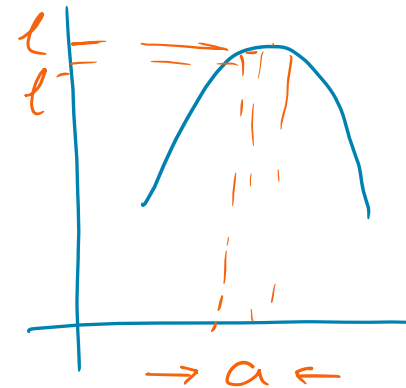
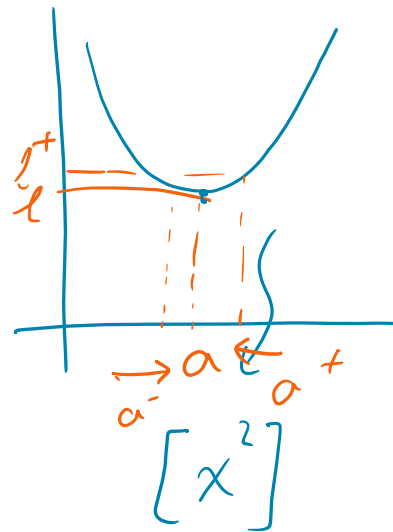
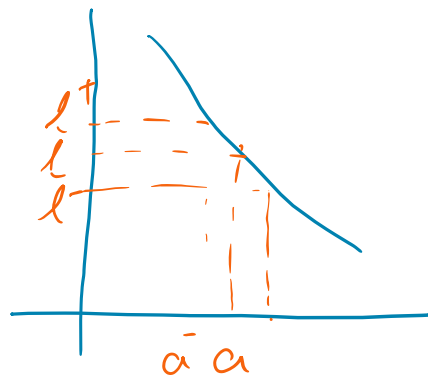
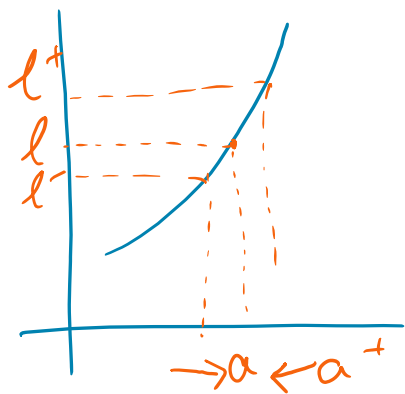
$$\lim_{x \rightarrow 0^-} f(x^3 - x) = +1$$

$$\lim_{x \rightarrow 3} [x] = \begin{cases} x \rightarrow 3^+ \Rightarrow \lim f(x) = 3 \\ x \rightarrow 3^- \Rightarrow \lim f(x) = 2 \end{cases}$$

$$\text{, lim} \quad | \quad \lim_{x \rightarrow 3/2} f(x) =$$

$$\lim_{x \rightarrow a} [f(x)] = l$$

$$a \begin{cases} \in \mathbb{Z} \Rightarrow \lim f(x) \rightarrow \text{ندار؟} \\ \notin \mathbb{Z} \Rightarrow \lim f(x) = \text{حد دارد} \end{cases}$$



$$f(x) = [x^2] \quad \lim_{x \rightarrow 0} f(x) = \begin{cases} 0^+ \rightarrow 0 \\ 0^- \rightarrow 0 \end{cases}$$

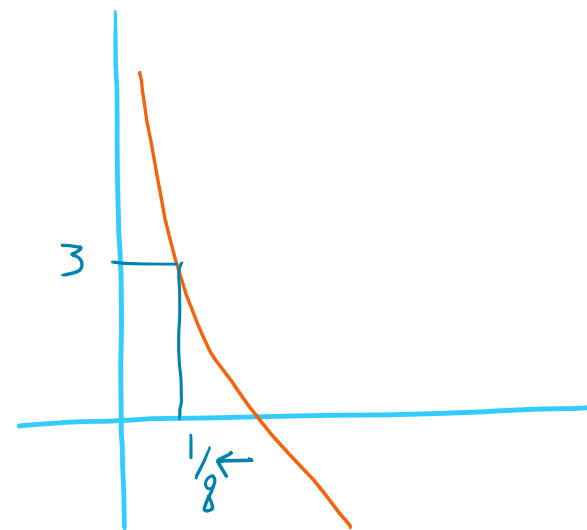
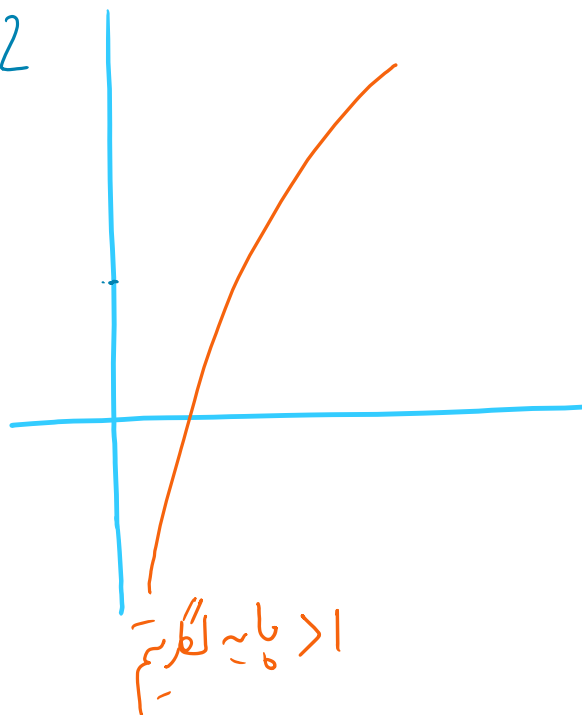
به طرک طهر زمانه که داخل بدترین مقدار جمع شود
تابع حد ندارد مگر آنکه در آن نقطه تابع بسته باشد

$(-2, 2)$
در چند نقطه حد ندارد

$$x^2 = k \Rightarrow x = \pm \sqrt{k} \quad k \in \mathbb{Z}$$

$$\{-\sqrt{3}, -\sqrt{2}, -\sqrt{1}, 0, \sqrt{1}, \sqrt{2}, \sqrt{3}\}$$

$$\lim_{x \rightarrow \frac{1}{8}^+} \left[\log^x \left(\frac{1}{2} \right) \right] = ? \quad [3^-] = 2$$



$$\begin{array}{c} 4^+ \\ \text{---} \\ 4^- \\ \text{---} \\ 1/2 \end{array}$$

$$\left[\lim_{x \rightarrow (\frac{\pi}{6})^-} 8 \sin x \right] = ? \quad 4$$

* تمرین

$$\lim_{x \rightarrow \frac{1}{5}} [5x] + [-5x] =$$

هنگام درگاه جدید تابع کسره، غرض کسره صاف باشد، جواب حد ندارد وجود ندارد است

انرا محدود ندارم

$$\lim_{x \rightarrow 3^+} \frac{x+1}{[x-3]} = ? \quad \checkmark$$

$$\lim_{x \rightarrow 0} \frac{1 + \sin x}{[x^2]}$$

$$\lim_{x \rightarrow 1} \frac{1 - [x]^2}{1 - [x]} = ? \quad \begin{cases} x \rightarrow 1^+ & \text{, line} \\ x \rightarrow 1^- & 1 \end{cases}$$

$$\lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3} [x] = ? \quad \frac{-(x-3)}{x-3} \underbrace{[3^-]}_2 = -2$$

$$\lim_{x \rightarrow -1^+} \frac{|x|}{[x]} \operatorname{sgn}(x+1) = ? \quad -1$$

$$f(x) = \left[\frac{4x^2 + \overbrace{3+1}^4 - 1}{x^2 + 1} \right] = \left[\frac{4(x^2 + 1) - 1}{x^2 + 1} \right] = \left[\underbrace{4}_{\text{تابع در تمامی نقاط حد دارد}} - \frac{1}{x^2 + 1} \right] = 4 + \underbrace{\left[\frac{-1}{x^2 + 1} \right]}_{-1} = 3$$

$x^2 \geq 0 \quad x^2 + 1 \geq 1 \Rightarrow 0 \leq \frac{1}{x^2 + 1} \leq 1 \Rightarrow -1 \leq \frac{-1}{x^2 + 1} \leq 0$

$$\lim_{x \rightarrow 1^+} \frac{[x-1]}{x-[x]} = ?$$

$$\frac{0 \text{ مطلق}}{0 \text{ حاد}} = 0$$

$$\lim_{x \rightarrow 0} \frac{[x^2] - x^2}{x^2} = ?$$

$$\lim_{x \rightarrow 0^+} \frac{1 - 2 \cos x}{3^x - 1} = ?$$

$$\lim_{x \rightarrow \frac{\pi}{4}^+} \sqrt[4]{\cot x - 1}$$

$$f(x) = \frac{(3 - [x]) \sqrt{x^2 - 4x + 4}}{x^2 - 4}$$

$$\lim_{x \rightarrow 2} f(x)$$

صورت لری میبیم

$$\frac{0}{0}$$

$$\frac{\infty}{\infty}$$

$$\infty - \infty$$

$$0 \times \infty$$

$$1^\infty$$

$$0^0$$

$$\infty^0$$

$$\frac{0}{0} - 1$$

باید رفع ابهام
 1- حد بی نهایت ← مشتق
 2- حدت عامل منفرد کننده ✓
 3- هم ارز ✓

$$\lim_{x \rightarrow \pi} \frac{1 + \cos^3 x}{\sin x} = \frac{0}{0}$$

$$\lim_{x \rightarrow \pi} \frac{(1 + \cancel{\cos x})(1 + \cos^2 x - \cos x)}{(1 + \cancel{\cos x})(1 - \cos x)} = \frac{3}{2}$$

$$\lim_{x \rightarrow \pi^+} \frac{\sqrt{1 + \cos x}}{\sin x} = \frac{0}{-\frac{\sqrt{2}}{2}}$$

$$1 + \cos x = 2 \cos^2 \frac{x}{2}$$

$$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$\lim_{x \rightarrow \pi} \frac{\sqrt{2 \cos^2 \frac{x}{2}}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{|\cos \frac{x}{2}| \times \sqrt{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{-\sqrt{2} \cancel{\cos \frac{x}{2}}}{2 \sin \frac{x}{2} \cancel{\cos \frac{x}{2}}}$$

$$\Rightarrow \lim_{x \rightarrow \pi} \frac{-\sqrt{2}}{2 \sin \frac{x}{2}} = -\frac{\sqrt{2}}{2}$$

$$\lim_{x \rightarrow 1^+} \frac{(x-1)\sqrt{x-1} + x^2 - 1}{x^3 - 1} = \lim_{x \rightarrow 1^+} \frac{(x-1)\sqrt{x-1} + (x-1)(x+1)}{(x-1)(x^2 + 1 + x)} = \lim_{x \rightarrow 1^+} \frac{\cancel{(x-1)}[\sqrt{x-1} + x+1]}{\cancel{(x-1)}(x^2 + 1 + x)} = \frac{2}{3}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{\sin^2 x (1 - \sin x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{(1 - \sin x)(1 + \sin x)}{\sin^2 x (1 - \sin x)} = \frac{2}{1} = 2$$

$\nearrow 1 - \sin^2 x$

$$\underbrace{f(x) \sim g(x)}_{x=a \text{ limit}} \Rightarrow \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = 1$$

$$x \rightarrow 0 \quad \left\{ \begin{array}{l} \sin x \sim x \\ \tan x \sim x \\ \cos x \sim 1 - \frac{x^2}{2} \Rightarrow \boxed{1 - \cos x \sim \frac{x^2}{2}} \end{array} \right.$$

همانطور

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = ? \quad \frac{0}{0} \Rightarrow \lim_{x \rightarrow 0} \frac{x}{x} = 1$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(\overset{u}{\cot x})}{\cot x} = \frac{0}{0} \Rightarrow \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x}{\cot x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x \cdot \sin 2x \cdot \sin 3x}{3x^2 \sin x} = \frac{x \cdot 2x \cdot 3x}{3x^2 \cdot x} = 2$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{1 - \cos 2x} = \lim_{x \rightarrow 0} \frac{x^2}{\frac{(2x)^2}{2}} = \frac{1}{2}$$

$1 - \cos x \sim \frac{x^2}{2}$

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\sin^2 x} = -2$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\tan^2 \frac{x}{2}} = 8$$

$$1 - \cos u \sim \frac{u^2}{2} \Rightarrow \lim_{x \rightarrow 0} \frac{\frac{(2x)^2}{2}}{\left(\frac{x}{2}\right)^2} = 8$$

$$\tan v \sim v$$

$$\frac{(x - \frac{\pi}{4})^2}{2}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \cos \left(x - \frac{\pi}{4}\right)}{\tan^2 \left(x - \frac{\pi}{4}\right)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{2}{2}}{\left(x - \frac{\pi}{4}\right)^2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{x^2} = ? \quad \frac{1}{2}$$

$$\lim_{x \rightarrow 1} \frac{1 - x^2}{(\arccos x)^2} = ? = \lim_{\alpha \rightarrow 0} \frac{1 - \cos^2 \alpha}{\alpha^2} = \frac{\sin^2 \alpha}{\alpha^2} = \frac{\alpha^2}{\alpha^2} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{x^2} = 4$$

$$\arccos x = \alpha$$

$$\cos \alpha = x$$

$$\boxed{\cos u \sim 1 - \frac{1}{2}u^2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{(1 - \tan x)x} = \lim_{x \rightarrow 0} \frac{1 - (1 - \frac{3x^2}{2})}{(1 - x)x} = \frac{\frac{3x}{2}}{1 - x} = \lim_{x \rightarrow 0} \frac{3x}{2(1 - x)} = 0$$

$$\lim_{x \rightarrow 0} \frac{x^3 - \sin^3 x}{x^5} = ?$$

بطاكر لكون

$$\left\{ \begin{array}{l} \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\ \tan x = x + \frac{x^3}{3} \\ \cos x = x - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \end{array} \right.$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$\lim_{x \rightarrow 0} \frac{x^3 - \sin^3 x}{x^5} = ? \quad \frac{\overbrace{x - \frac{x^3}{6}}^{x - \frac{x^3}{6}} \underbrace{(x^2 + \sin^2 x + x \sin x)}_{\sim x^2 + x^2 + x^2}}{x^5} = \frac{(\cancel{x} - \cancel{x} + \frac{x^3}{6})(3x^2)}{x^5} = \lim_{x \rightarrow 0} \frac{\frac{1}{2} x^5}{x^5} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2} \quad \lim_{x \rightarrow 0} \frac{x + \frac{x^3}{3} - (x - \frac{x^3}{6})}{x^3} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\sin^3 x - x^3}{\tan^9 x} = -\frac{1}{6}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x - x^2 \cos^2 x}{x^2 \sin^2 x} = \frac{2}{3}$$

$$\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5} = ? \quad \frac{1}{120}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$$

$$\cos x = x - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

* مهم

$$\lim \frac{\sinh x - \tan x}{x(e^{x^2} - 1)} =$$

$$\lim \frac{e^x - \sin x + \cos x - 2}{\sqrt[3]{1+x^2} - 1} =$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - \sin x}{x \sin x} = \frac{1}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$